3) It is perfectly

Isothermal process :-

In isothermal process is 1 in which the temperature of the system does not change i.e. $dT=0$ i.e there is no change in internal energy of the system ie $du=0$.

From First Law of Thermodynamics

$$\boxed{dQ = du+dW} \longrightarrow ①$$

$$dQ = dW$$

Eq ① for isothermal process

Hence during isothermal expansion of gas work is done by the gas which is equal to the amount of heat absorbed by it. Similarly, during isothermal compression of gas work is done on the gas which is equal to the amount of heat rejected by it.

Adiabatic Process:-

An Adiabatic Process is one which takes place in such a manner that no heat enters or leaves the system i.e $dQ=0$

Hence from first law of thermodynamics

$$0=du+dw$$

$$\boxed{du=-dw} \longrightarrow ② \text{ for adiabatic process.}$$

During an adiabatic compression of gas work is done on the gas (-ve) and the internal energy increases by an amount which is equal to the workdone on the gas. Similarly during adiabatic expansion of gas work is done by the gas (+ve) and hence the internal energy decreases by the amount equal to the workdone by the gas.

Reversible and Irreversible Processes:-

Reversible Process:-

A reversible process is one which can be reversed in such a way that all changes occuring in the direct process or exactly repeated in the opposite order and inverse sense no changes are left in any of the bodies taking part in the process are in the surroundings.

Conditions for Reversibility:-

- * Their must be complete abeence of forces such as friction, viscosity, electric resistance etc.
- * The direct and reverse processes must take place infinetly slow.
- * The temperature of the system must not differ in surroundings.

1) Examples all isothermal and adiabatic changes are reversible if they are performed very slow.

2) When a certain amount of heat is absorbed by ice it melts. If the same amount of heat is removed from it the water is formed in direct

process will be converted into eyes.

3) And extremely slow extension or contraction of a spring without setting of oscillations.

4) Very slow evaporation or condensation.

It is only an ideal concept.

Irreversible Process:

Any process which is not reversible exactly is an irreversible process. All natural processes such as conduction, radiation, radio active decay etc are irreversible.

Examples:

1) Sudden expansion or contraction and rapid evaporation or condensation are the examples.
2) Joule Thomson Effect.
3) Heating produced by the passage of an electric current through resistance.
4) Heat transfer between bodies at different temperatures.

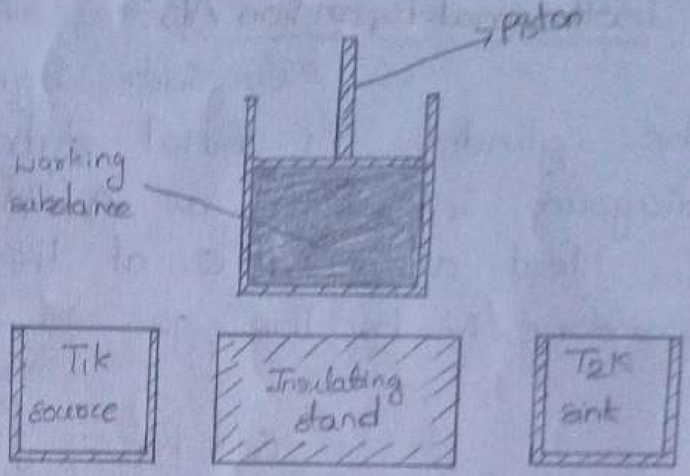
Theorem :

UNIT-II

Thermodynamics

A heat engine is a device to convert heat into work. In the year 1824. French engineer carnot deviced a practical engine as shown in fig. It consists of the following parts.

1. The working substance
2. The source of heat
3. The sink
4. Insulating stand



1. **The Working Substance** :- The working substance is an ideal gas enclosed in a cylinders -piston arrangement

2. **The Source of heat** :- A hot body of high thermal capacity maintained at a high temperature. T_1K serves as a source.

3. **The Sink** :- A cold body maintained at a lower temperature T_2k serves as a sink. This is surrounding atmosphere.

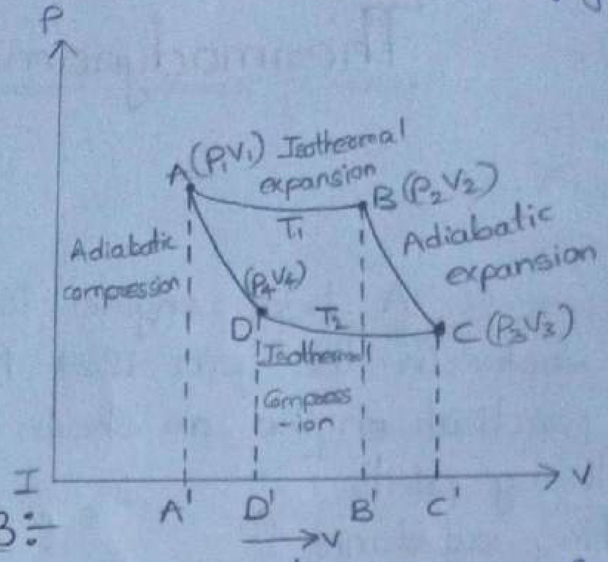
4. **Insulating stand** :- To make the hole system perfectly a non-conducting stand is put on the base of the cylinder.

Carnot's Cycle :-

The working substance is made to undergo a cyclic operation made up of four parts.

1) Isothermal expansion
2) Adiabatic expansion
3) Isothermal compression
4) Adiabatic compression

All the four operations are shown in fig②.



fig②

1. Isothermal Expansion AB :-

Consider 1 gm mole of gas is contained in the cylinder. It's initial state is denoted by A in indicator diagram in fig②, as A (P_1V_1) is kept on source which is at T_1. Heat absorbed Q_1 at this time and the amount of workdone W_1 by the gas is at $B(P_2V_2)$. Then

$$Q = W_1 = \int_{V_1}^{V_2} P\,dv = \int_{V_1}^{V_2} RT_1\left(\frac{dv}{v}\right)$$

$$= RT_1 \log\left(\frac{V_1}{V_2}\right) = \text{area of } ABB'A'A \rightarrow ①$$

2. Adiabatic Expansion BC :-

The cylinder is detached from the source and is placed on insulating stand piston is allowed to move adiabatically tild the temperature of gas is same as that of sink (T_2k). The expansion is shown by BC and the gas is at P_3V_3. Then the workdone by the gas is given by

$$W_2 = \int_{V_2}^{V_3} P\,dv = \int_{V_2}^{V_3} K \frac{dv}{V^{\gamma}}$$

$$\rightarrow K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_2}^{V_3} \Rightarrow \frac{P_3V_3 - P_2V_2}{1-\gamma}$$

$$\Rightarrow \frac{RT_2 - RT_1}{1-\gamma}$$

$$W_2 = \frac{R(T_2-T_1)}{\gamma-1} = \text{area of } BCC'B'B \longrightarrow ②$$

After this stage the pressure of the gas is much reduced and it has lost its capacity of doing work, so now gas is brought back to its initial state (P_1V_1) by following isothermal compression and adiabatic compression.

3) Isothermal Compression CD :- The cylinder is separated from the insulating stand and placed on the sink at T_2. The gas is compressed very slowly till the gas attains the state D at (P_4V_4) respectively. It is shown by curve CD. The quality of heat rejected to the sink be Q_2.

This is equal to the workdone W_3 on the gas

$$Q_2 = W_3 = \int_{V_3}^{V_4} PdV = RT_2 \int_{V_3}^{V_4} \frac{dV}{V}$$

$$= RT_2 \log\left(\frac{V_4}{V_3}\right)$$

$$= -RT_2 \log\left(\frac{V_3}{V_4}\right)$$

$$= \text{Area of } CC'D'DC \longrightarrow ③$$

4) Adiabatic Compression DA :- This is final state of the process when the cylinder is detached from the sink and placed on the insulating stand. The gas compressed and it undergo a slow adiabatic compression till the state A is again reached. Now the temperature rises from T_2k to T_1K and it is indicated by the curve DA on the indicator diagram. The workdone W_4 on the gas during this process will be

$$W = \int_{V_4}^{V_1} PdV = -\frac{R(T_1-T_2)}{\gamma-1} = \text{area of } DD'A'AD \longrightarrow ④$$

from ② & ④ $W_2 = W_4$

Efficiency of the Engine :-

In order to calculate the efficiency of heat engine, we should calculate the workdone and the amount of heat taken. Let Q_1 be the heat tak during isothermal expansion at temperature T_1k & Q_2 b the quantity of heat rejected at temp T_2k to the si

$$\eta = \frac{\text{Workdone by the engine}}{\text{heat taken from source}}$$

$$\eta = \frac{\text{Heat Converted into work}}{\text{Heat taken}}$$

or $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

(or) In terms of temperature from equation

$$\eta = \frac{W}{Q_1} = \frac{R(T_1 - T_2)\log\left(\frac{V_2}{V_1}\right)}{RT_1 \log\left(\frac{V_2}{V_1}\right)}$$

$$\eta = \frac{T_1 - T_2}{T_1} \Longrightarrow n = 1 - \frac{T_2}{T_1} \longrightarrow ⑥$$

Impracticability of Carnot's Engine :-

1) In Carnot's Engine that the heat is to be taken and rejected strictly temperature which is practically very difficult.

2) The engine has very slow working. It may of no use in day-to-day life.

3) It is perfectly reversible in its ACTION.

process will be converted into eyes.
3) And extremely slow extension or contraction of a spring without setting of oscillations.
4) Very slow evaporation or condensation.
It is only an ideal concept.

Irreversible Process:-

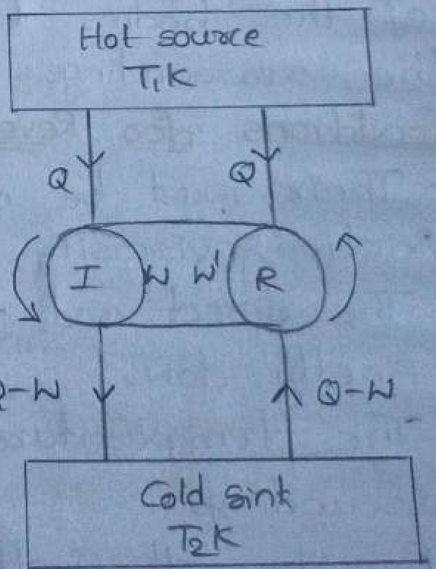
Any process which is not reversible exactly is an irreversible process. All natural processes such as conduction, radiation, radio active decay etc are irreversible.

Examples:-
1) Sudden expansion or contraction and rapid evaporation or condensation are the examples.
2) Joule Thomson Effect.
3) Heating produced by the passage of an electric current through resistance.
4) Heat transfer between bodies at different temperatures.

Carnot's Theorem:-

According to this theorem no engine can be more efficient that reversible engine working b/w the same two temperatures.

It also stated as all reversible engines working between the same two temperatures have the same efficiency whatever may be the working substance.



To prove this let us consider a reversible engine R and irreversible engine I working with the same source and sink. Here I works in the forward direction while R works in the backward direction.

Let engine I absorb an amount of Q from source, convert a part of. it W into work and transfer the balance Q-W to the sink, lly let the engine R absorb an amount of heat Q' from the sink a certain amount of work W' is done on that working substance and transfers the net amount $Q'+W'$ to the source.

Here we assume that the amount of heat which I receives from the source is returned by R to the source. Hence $Q'+W'=Q$

or $W'=Q-Q'$

If possible, let the reversible engine I be more efficient than reversible engine R ie.

efficiency of I > efficiency of R

$$\frac{W}{Q} > \frac{W'}{Q} \quad \text{i.e} \quad W > W' \longrightarrow ①$$

Now imagine that two engines are coupled by a belt in such a way that engine I works directly and it drives the engine R reversibly. The compound engine extracts an amount of heat $(Q-W')$ from the sink and gives an amount of heat from the sink $= (Q-W')-(Q-W) = W-W' \longrightarrow ②$

This is +ve according to eq ①

The above fact shows that by coupling engine the heat absorbed from the sink $(Q-W')$ is greater than heat given back to the sink $(Q-W)$, while may be taken from the sink, workdone and no change may be left with source.

This is impossible according to the 2nd law of thermodynamics which states that it is impossible for any device to extract heat from a simple reservoir and converting the whole into work. Thus no engine working between a source and sink can be more efficient than a reversible engine.

Thermodynamic Scale of temperature and its identity with perfect gas scale :-

In 1848 Kelvin started with the result that the efficiency of an reversible engine working b/w the same two temperatures is a function of two temperatures only and is independent of the nature of working substance. The ideal gas scale is also known as thermodynamic scale of temperature.

Let Q_1 be the amount of heat absorbed by a reversible heat engine at temp T_1 and Q_2 be the quantity of heat rejected at temp T_2 on the scale. The efficiency is only the function of the these two temperatures, therefore

$$\eta = 1 - \frac{Q_2}{Q_1} = f(T_1 T_2)$$

$$\frac{Q_2}{Q_1} = 1 - f(T_1 T_2)$$

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f(T_1 T_2)} = F(T_1 T_2) \longrightarrow ①$$

Where F denotes some new functions of T_1 and T_2.

Now we consider another reversible heat engine absorbing heat Q_2 at temperature T_2 and rejecting heat Q_2 at temperature T_3 then

$$\frac{Q_2}{Q_3} = F(T_2 T_3) \longrightarrow ②$$

lly for

temperatures T_1 and T_3 $(T_1 > T_3)$ we have

$$\frac{Q_1}{Q_3} = F(T_1 T_3) \longrightarrow ③$$

Multiplying eq ① and eq ② we get

$$\frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} = F(T_1 T_2) \times F(T_2 T_3) \longrightarrow ④$$

Comparing eq ③ and eq ④ we have

$$F(T_1 T_3) = F(T_1 T_2) \times F(T_2 T_3) \longrightarrow ⑤$$

It is observed from eq ⑤ that LHS has no T_2. This is only possible if we choose the following form of these functions:

$$F(T_1 T_2) = \frac{\phi(T_1)}{\phi(T_2)}$$

$$\text{and } F(T_2 T_3) = \frac{\phi(T_2)}{\phi(T_3)} \longrightarrow ⑦$$

Where ϕ is another function of temperature.

Now $F(T_1 T_2) \times F(T_2 T_3) = \frac{\phi(T_1)}{\phi(T_2)} \times \frac{\phi(T_2)}{\phi(T_3)} = \frac{\phi(T_1)}{\phi(T_3)} = F(T_1 T_3)$

Thus eq ⑤ is satisfied.

From eq ① we get $\frac{Q_1}{Q_2} = \frac{\phi(T_1)}{\phi(T_2)} \longrightarrow$ ⑧

Since $Q_1 > Q_2$, $\phi(T_1) > \phi(T_2)$ provided $T_1 > T_2$.

Thus $\phi(T)$ is a linear function of T because $\phi(T)$ has a greater value for greater value of T.

Let $\phi(T) = \tau$ we have

$$\frac{Q_1}{Q_2} = \frac{\phi(T_1)}{\phi(T_2)} = \frac{\tau_1}{\tau_2} \longrightarrow ⑨$$

eq ⑨ defines Kelvin Scale of temperature. The thermodynamic or Kelvin Scale of temperature is that scale, the ratio of any two temperatures is the same as the ratio of the heat absorbed and rejected by reversible carnot engine operating b/w these two temperatures.

<u>It's identity with perfect gas</u>:-

The equality of kelvin temp and ideal gas temp is shown below.

Let T_1 be the temperature of hot reservoir (source) ~~and~~ and T_2 be the temperature of cold reservoir (sink) as measured on kelvin scale. The efficiency of carnot's reversible heat engine working b/w two temperatures is given by

$$\eta = 1 - \frac{\tau_1}{\tau_2} \longrightarrow ⑩$$

Now denoting the temperatures of source and sink by T_1 and T_2 on ideal gas scale then the efficiency of carnot engine is exposed as

$$\eta = 1 - \frac{T_2}{T_1} \longrightarrow ⑪$$

Comparing ⑩ & ⑪ we get

$$\frac{\tau_2}{\tau_1} \text{(Kelvin Scale)} = \frac{T_2}{T_1} \text{(ideal gas scale)}$$

Let τ represents any temp on kelvin scale and T refers to any temp on ideal gas scale then

$$\boxed{\tau = T} \longrightarrow ⑫$$

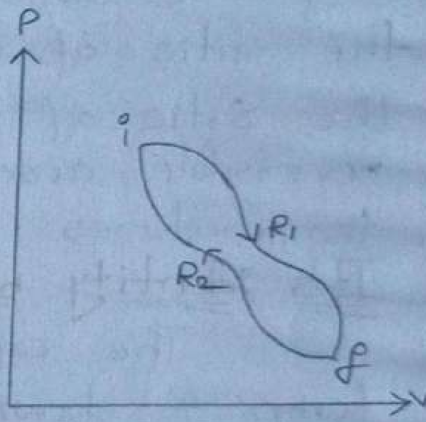
Thus, the kelvin temperature τ and ideal gas temperature T are numerically equal to each other.

Entropy:-

This is a physical quantity like P,V,T. The entropys is also a thermodynamic variable (coordinate) of the system. The entropy of the system is a measure of the system its degree of disorder.

The changes of entropy can be calculated as the ratio of amount of heat (dQ) taken of to the absolute temperature (t) ie $ds = \frac{dQ}{T}$.

Entropy - Joule/Kelvin.



As shown in figure le the process takes place from initial state i to final state f via path R_1. let the process again starts from final state f and reach initial state i via different path R_2 as this process is cyclic we have from Clausius theorem

$$\oint_R \frac{dQ}{T} = 0$$

$$\int_f^i \frac{dQ}{T} = 0 \longrightarrow ①$$

If S_i and S_f are the initial entropy and final entropy of the system then

$$S_f - S_i = \int_i^f \frac{dQ}{T} \longrightarrow ②$$

Where the function $\int_i^f \frac{dQ}{T}$ is defined as entropy of the system.

Entropy - When ice changes into steam :-

Let us consider the melting point of ice be T_1 and the boiling water point T_2. Considering a mass m of ice it requires latent heat mL_1 to get converted into water at T_1°C. Where L_1 is Latent heat of fusion.

1> Change in entropy when a mass m of ice melts into water at T_1 is

$$\Delta S_1 = \frac{mL_1}{T_1}$$

Assuming that it is at constant volume $V_f = V_i$ so that $\log_e\left(\frac{V_f}{V_i}\right) = 0$

2> Then change in entropy when water get heated from T_1 to T_2 is $\Delta S_2 = mc\log_e\left(\frac{T_2}{T_1}\right)$

Here c is specific heat capacity of water

3> Change in entropy when water at T_2 is converted into steam at the same temperature is

$$\Delta S_3 = \frac{mL_2}{T_2}$$

Where L_2 is Latent heat of vapourisation of water.

The total change in entropy when m mass of ice is converted into steam is given by

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3$$

$$= \frac{mL_1}{T_1} + mc\log_e\left(\frac{T_2}{T_1}\right) + \frac{mL_2}{T_2}$$

$$= m\left[\frac{L_1}{T_1} + c\log_e\left(\frac{T_2}{T_1}\right) + \frac{L_2}{T_2}\right]$$

Entropy and Disorder :-

Work involves an orderly motion, heat involves a disorderly motion of the molecules. The more temperature of a gas the more will be the disorderly motion.

Whenever work is converted into internal energy of a system the disorderly motion of molecules will increase. That is there is a transition from order to disorder.

When temperature of gas in container is raised the disorderly motion of molecules also increase.

Thus in all natural processes, the disorder increases or there is a tendency on the part of the nature to proceed towards a state of more disorder.

The entropy is a measure of the degree of molecular disorder. According to Boltzmann the entropy S and disorder W are related as

$$\boxed{S = K\log_e W} \longrightarrow ①$$

Where K is Boltzmann Constant.

With the help of this equation we can understand about entropy of a system in non-equilibrium states also.

Hence A non-equilibrium state corresponds to a certain degree of disorder and hence to definite value of entropy

Entropy of Universe:-

In a reversible process the entropy of the system and its surroundings remains the same. The processes that are taking place in nature are all irreversible processes. The entropy of a particular system may decrease in entropy. Hence we conclude that the entropy of the universe always increases.

Thus while the energy of the universe remains constant according to the law of conservation of energy. The entropy does not obey conservation law and entropy of the universe always increases.

In this universe 1) There is always a decrease in the available energy in the universe with the passage of time and

2) There is always an increase in the entropy of the universe with the passage of time. The above two are related to by the equations,

Loss of available energy (Q_2) = $T_2 \Delta S$ increase in entropy.

Entropy for reversible and irreversible processes:-

Irreversible Process:- The equation $S_f - S_i = \int_i^f \frac{\delta Q}{T}$ is to calculate entropy change of the system in order to find out change in entropy for any irreversible process b/n two given equilibrium states.

For an reversible engine operating b/n two temps

$$\eta_R = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

For an irreversible engine

$$\eta_I = 1 - \frac{Q_2}{Q_1}$$

For Carnot's theorem we have

$$\boxed{\eta_I > \eta_R}$$

$$1 - \frac{Q_2}{Q_1} > 1 - \frac{T_2}{T_1}$$

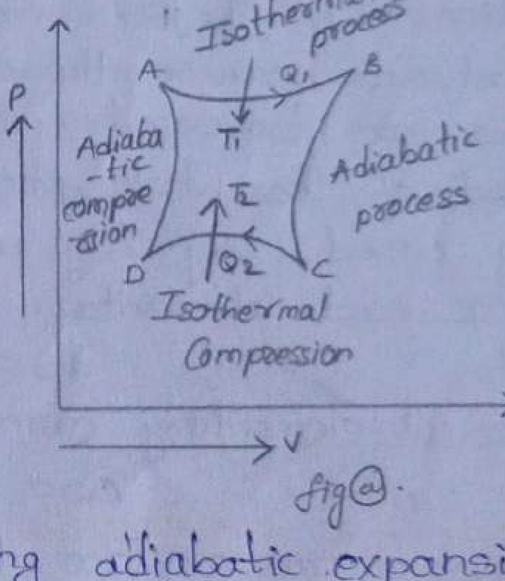
$$\frac{Q_2}{Q_1} > \frac{T_2}{T_1}$$

$$\cancel{\frac{Q_2}{Q_2}} \frac{Q_2}{T_2} - \frac{Q_1}{T_1} > 0$$

$$\boxed{\int \frac{\partial Q}{T} > 0}$$

i.e $\int \frac{\partial Q}{T} > 0$ for irreversible process.

<u>Reversible Process</u>:-



fig(a).

Let us consider a change in entropy of a substance when it is taken through a reversible carnot cycle ABCD as shown in fig(a). Start from initial state A the working substance undergoes an isothermal expansion AB absorbing an amount of heat Q_1 from the source at temp T_1. Increase in entropy $= \frac{Q_1}{T_1}$. During adiabatic expansion BC, no heat is taken or given up by the working substance, Hence there is no change in entropy.

During isothermal compression CD, the working substance rejects a quantity of heat Q_2 at T_2. Hence the decrease in entropy = Q_2/T_2.

Finally, during the adiabatic compression DA, entropy of the working substance remain constant.

Thus the net change in entropy of the working substance in cycle ABCDA is given by

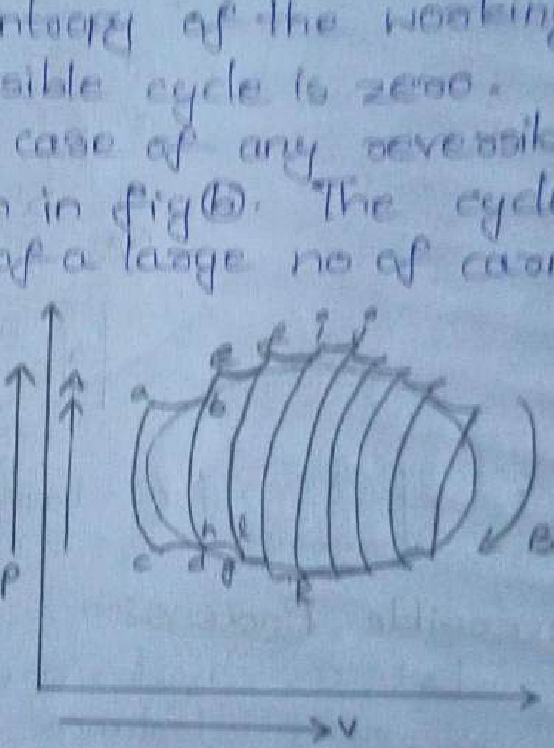
$$\Delta S = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

But we know that $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$ {By Kelvin Scale of temp}

$$\boxed{\therefore \Delta S = 0}$$

Hence the total change in entropy of the working substance in a carnot's reversible cycle is zero.

Now take a general case of any reversible cycle A ⟶ B ⟶ A as shown in fig(b). The cycle may be considered to be made up of a large no of carnot cycles like abcd, efgh, ijkl.



Now imagine that the substance taken around the smooth curve A ⟶ B ⟶ A is taken through elementary carnot cycles.

The effect of the portions bh, fl, jp ... etc cancel out because these paths are traversed twice in the reverse order. The net effect of the whole process is that the substance is taken along closed zig-zag path of abefi ... gheda.

For each elementary carnot cycle we have

$$dS = 0$$

For all elementory carnot cycles, taking the sum, we have

$$\Delta S = \sum \frac{dQ}{T} = 0$$

Now we can replace the sum by an integral for the whole close cycle

$$\Delta S = \oint \frac{dQ}{T} = 0$$

So the net change in entropy is zero for any reversible cycle. This statement is also called as Clausius theorem. The clausius theorem may be stated as the sum of quantities of heat transfer during isothermal change divided by absolute temperature of the isothermal in a reversible cycle is zero.

Temperature - Entropy diagram (TS Diagram) :

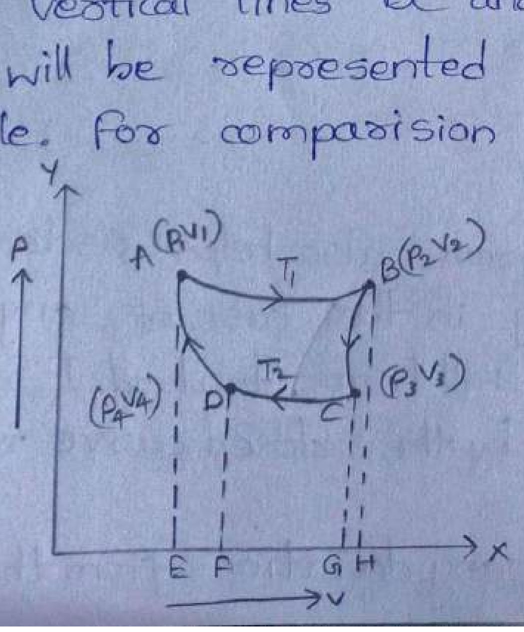
This equation is analogous to the workdone (w)

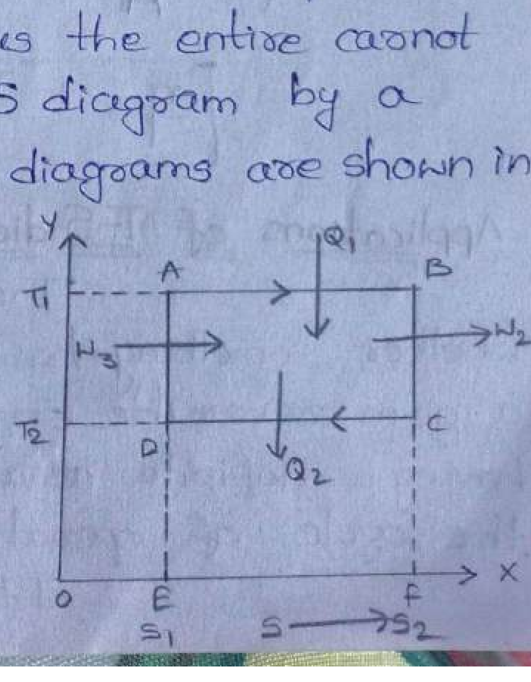
$$\therefore W = \int P dv$$

Where dv is the change in volume on applying a pressure p It is just as the area under the curve of P and V as in PV diagram gives the workdone.

A graph obtained by plotting the temp T on Y-axis and entropy S on the X-axis is called a T-S diagram. (also called tephigram). The thermodynamic state of the working substance is determined by this T-S diagram.

A T-S diagram displays the characteristics of a carnot cycle better than the P-V diagram. The isothermal process will be represented by two horizontal lines AB and CD while the two adiabatic processes will be represented by two vertical lines BC and AD. Thus the entire carnot cycle will be represented on the TS diagram by a rectangle. For comparision the two diagrams are shown in fig.





AB represents isothermal expansion during which temp remain at T_1, but entropy S_1 to S_2. BC represents adiabatic expansion during which temp falls from T_1 to T_2 but no change in entropy. CD- isothermal compression at constant T and entropy S_2 to S_1.

DA represents adiabatic compression during which the temperature rises from T_2 to T_1 with constant S_1. Heat absorbed at isothermal expansion AB is

$$Q_1 = T_1(S_2 - S_1)$$

from $Q_1 = T.\Delta S$ = area of ABFE

Heat rejected by the working isothermal compression CD is

$$Q_2 = T_2(S_2 - S_1) = \text{Area of DCFE.}$$

∴ The available energy converted into useful work will be

$$W = Q_1 - Q_2$$

$$= \text{Area of ABFE} - \text{Area of DCFE}$$

$$= \text{Area of ABCD.}$$

Thus the area of cycle on the T-S diagram gives us an estimate of the available thermal energy that can be converted into useful work.

Now the efficiency of the heat engine is given by

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{\text{Area of ABCD}}{\text{Area of ABFE}} = \frac{AB \times AD}{AB \times AE}$$

$$\eta = \frac{AD}{AE} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

$$\boxed{\eta = 1 - \frac{T_2}{T_1}}$$

Applications of T-S diagram :-

The T-S diagram also helps us to know whether something has gone wrong in the case of engine. For a given reversible engine there will be a standard T-S diagram, having a definite area enclosed by the closed curve representing the cycle of operations.

If there is any deviation from this standard

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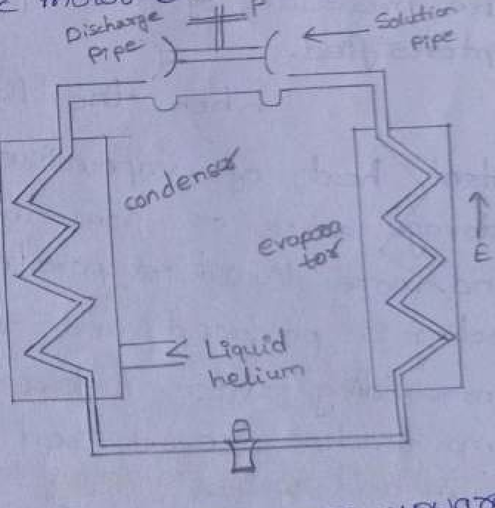
diagram and if the area decreases then it is an indication of some irreversible processes have kept into and are to be checked out to convert a possible break down to loss of efficiency. The work value of the fuel employed in an engine can also be as certained with the help of a T-S diagram.

Working of refrigerator:-

A machine used for producing low temp below surrounding and maintaining an enclosure at the temp is called as refrigerator. Refrigerator is a carnot engine working in backward direction. Refrigerating machines are of two types.

1. Vapour compression machines and (frigidairie)
2. Vapour absorption machine (refrigerator)

The vapour compression machine refrigerator is shown in fig. The refrigerant is compressed by a pump P driven by an electric motor (work as compressor). There were two values V_1 and V_2 at the bottom of compression pump P. The valve V_2 is for suction of low pressure vapour from evaporator E and the valve V_1 is for allowing the compressed vapour to let into the condenser C.



When the piston of the pump ↑ moves upwards the pressure in the cylinder falls below the valve V_1 remains closed. The low pressure vapour is sucked into cylinder.

When the piston ↓ is pushed downwards the pressure of the vapour in cylinder increases and the valve V_1 open while V_2 remains closed. Now the vapour under high pressure enters into the condensor coil C.

The condensor coil C is surrounded by enclosure in which cold water is circulating. The heat of compression is removed by the cold water. So the vapour under high pressure and low temp liquefies in C.

The regulator valve reduces the pressure of liquid from high valve to low valve. Due to low pressure the liquid evaporates in E. The evaporation of liquid under reduced pressure is a cooling process.

The reason is that when a substance changes from liquid state to vapour phase, it is with absorption of heat.

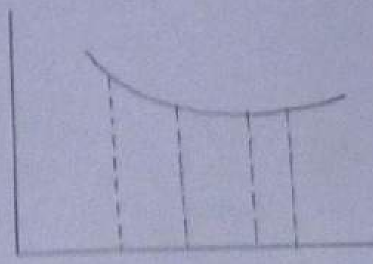
If heat is not taken from any external source, the heat for phase changes is taken from liquid itself which cools. If the evaporation is promoted by reducing the pressure on the liquid surface the cooling is intensified.

When the liquid evaporates in E, it extracts latent heat of vapourisation from surrounding cold storage space or from the surrounding liquid. As more and more liquid evaporates in evaporator coil, more cooling is produced and the temp of storage space or surrounding water is lowered. After a certain time, the temp reaches the desired valve. The cycle is repeat again and again.

Workdone in an isothermal process:

When an ideal gas expands isothermally work is done by the gas against the surrounding and heat flows from outside into the gas.

When the gas is compressed isothermally, work is done on the gas and heat flows out from the gas.



Let one gram mole of an ideal gas expanded isothermally. As shown in fig. Let P_i and V_i be the pressure and volume at initial state A and P_f and V_f be the pressure and volume at final state.

Let P and V be the pressure and volume at any intermediate state C. Let assume that pressure P remains constant for a small increase in volume dv. The workdone by the gas during this small expansion is Pdv, shown in fig (shaded).

The net workdone by the gas when it expands from V_i to volume V_f is given by

$$W = \int_{V_i}^{V_f} P\,dv$$

For isothermal process PV

$$PV = RT \quad \text{or} \quad P = \left(\frac{RT}{V}\right)$$

$$W = \int_{V_i}^{V_f} RT\left(\frac{dv}{V}\right)$$

$$W = RT\log_e\left(\frac{V_f}{V_i}\right)$$

$$W = 2.3026\ RT\log_e\left(\frac{V_f}{V_i}\right)$$

The workdone is equal to the area bounded by the curve and the volume axis.

<u>Workdone in an adiabatic process:</u>

Let one gram mole of an ideal gas expand adiabatically from initial state A to final state B as shown in fig during adiabatic expansion both pressure and volume change.

Let P and V be the pressure and volume at intermediate state C for small change in volume dv from C to D. Let pressure remains constant. The workdone by gas during the small expansion is Pdv. This is equal to CDD'C' as shown in fig.

The net workdone by the gas during expansion from V_i to V_f is given by $W = \int_{V_i}^{V_f} P\,dv$

For an adiabatic change $PV^{\gamma} = K$ (constant)

$$P = \frac{K}{V^{\gamma}}$$

$$W = \int_{V_i}^{V_f} \left(\frac{K}{V^{\gamma}}\right) dV$$

$$= K \int_{V_i}^{V_f} V^{-\gamma} dV$$

$$= K \left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f}$$

$$= \frac{K}{1-\gamma}\left[V^{1-\gamma}\right]_{V_i}^{V_f}$$

$$= \frac{K}{1-\gamma}\left[V_f^{1-\gamma} - V_i^{1-\gamma}\right]$$

$$= \frac{1}{1-\gamma}\left[\frac{K}{V_f^{1-\gamma}} - \frac{K}{V_i^{\gamma-1}}\right] \quad \left[\because P_i V_i^2 = P_f V_f^{\gamma} = K\right]$$

$$= \frac{1}{1-\gamma}\left(\frac{P_f V_f^{\gamma}}{V_f^{\gamma-1}} - \frac{P_i V_i^{\gamma}}{V_i^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\left[P_f V_f - P_i V_i\right]$$

$$W = \frac{1}{1-\gamma}\left[P_f V_f - P_i V_i\right]$$

When the gas is compressed the workdone is given by

$$W = \frac{1}{1-\gamma}\left(P_i V_i - \cancel{\gamma} P_f V_i\right)$$

$$= \frac{1}{\gamma-1}\left(P_f V_f - \cancel{\gamma} P_i V_i\right)$$

If the volume are V₁ and V₂ and pressure are P₁ and P₂ then the workdone under adiabatic compression is given by

$$W = \frac{1}{\gamma-1}\left[P_2 V_2 - P_1 V_1\right]$$

If T_1 and T_2 are temp of initial and final states, we can also write

$$W = \frac{R}{1-\gamma}\left[T_f - T_i\right]$$

$$W = \frac{R}{\gamma-1}\left[T_i - T_f\right]$$

A in

Unit - IV

Low Temperature Physics.

Questions:

① Joule Thomson - Effect.

② Distinction b/w Joule Thomson expansion and adiabatic expansion, Joule kelvin expansion.

③ Joule Thomson effect - Expression for cooling

④ Liquification of Helium

⑤ Principle of Refregiration

⑥ kapitza's method

⑦

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Joule - kelvin Effect (Porous Plug Experiment)

The principle of Joule kelvin experiment is shown in fig. PQ is thermally insulated cylinder containing porus plug, and fitted with two non conducting pistons A and B. The porus plug separated the Cylinder into two parts. The gas is compressed to a high pressure P_1 on one side, the gas then passes through the porous plug on the other side. There is a constant lower pressure P_2. When the compressed gas molecules come towards the lower pressure side after passing through porous plug, they are drawn apart & hence suffer a change in temperature. The change in temperature is called as Joule-Thomson effect and is stated as "When a gas under a constant pressure is passed through a porous plug to a region of lower constant pressure, the gas suffers a change in temperature

Such a change was actually demostrated by Joule-and kelvin and hence known as Joule-kelvin or Joule Thomson effect.

Referring to fig suppose that the piston A is moved towards the porous plug. The gas it transferred from left to right compartment and is assumed that the high pressure on left and low pressure on right are kept constant.

Let P_1V_1 and T_1 represent pressure, volume, and temperature on high pressure side and P_2V_2, T_2 quantities on lower side. Let u_1 and u_2 the initial energies of unit mass of the gas before and after passing through porous plug.

Workdone on the gas by piston A = P_1V_1

Workdone by the gas on piston B = P_2V_2

∴ Networkdone by the gas = $(P_2V_2 - P_1V_1)$

As the system is thermally isolated, this work must have been done at the expense of the initial energy of the gas. Hence

$$P_2V_2 - P_1V_1 = u_1 - u_2$$

$$u_1 + P_1V_1 = u_2 + P_2V_2 = \text{Constant}$$

The quantity $u+Pv$ is known as enthalpy. Thus in Joule kelvin effect the initial and final enthalpies are equal i.e., enthalpy remains constant.

$$\boxed{u + PV = \text{Constant}}$$

Expression for Joule-Thomson Cooling:

Consider a thermally insulated cylinder PQ as shown in fig. divided into two compartments by a porous P_1; P_2 are constant pressures of both sides. Let V_1 and V_2 be the volumes of the gas before and after passing through the porous plug.

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External workdone on the gas by
the piston = P_1V_1

External workdone by the gas.
on the piston = P_2V_2

Net amount of workdone by the gas = $(P_2V_2 - P_1V_1)$

If we assume the gas to obey vander wall's equation the attraction b/w the molecules is equivalent to an internal pressure $\left(\frac{a}{v^2}\right)$ where a is a constant and v be the volume occupied by the gas. The internal workdone against these forces when 1g mole expands from volume v_1 to volume v_2 is

$$\int_{v_1}^{v_2}\left(\frac{a}{v^2}\right)dv = \frac{a}{v_1} - \frac{a}{v_2}$$

Thus the total workdone w is given by

$$W = (P_2V_2 - P_1V_1) + \frac{a}{V_1} - \frac{a}{V_2} \longrightarrow ①$$

According to vander wall's equation we have

$$\left(P + \frac{a}{v^2}\right)(v-b) = RT$$

$$Pv + \frac{a^2}{v^2}v - Pb - \frac{ab}{v^2} = RT$$

As a and b are small quantities $\frac{ab}{V^2}$ can be neglected

$$PV + \frac{a}{V} - Pb = RT$$

$$PV = RT - \frac{a}{V} + Pb$$

$$P_1V_1 = RT - \frac{a}{V_1} + P_1 b \text{ and } P_2V_2 = RT - \frac{a}{V_2} + P_2 b$$

$$\therefore P_2V_2 - P_1V_1 = a\left[\frac{1}{V_1} - \frac{1}{V_2}\right] - b(P_1 - P_2)$$

Substituting the values of $(P_2V_2 - P_1V_1)$ from eq ② in eq ①

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we get

$$w = a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) - b\left(P_1 - P_2\right) + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = 2\left(\frac{a}{V_1} - \frac{a}{V_2}\right) - b\left(P_1 - P_2\right)$$ → ③

Since $\left(\frac{a}{V_1}\right)$ and $\left(\frac{a}{V_2}\right)$ are small quantities, we can use the approximate reaction.

$$PV = RT$$

$$V = \frac{RT}{P}$$

$$\frac{1}{V} = \frac{P}{RT}$$

$$\frac{a}{V} = \frac{Pa}{RT}$$

$$\frac{a}{V_1} = \frac{P_1 a}{RT} \text{ and } \frac{a}{V_2} = \frac{P_2 a}{RT}$$

Now eq③ becomes

$$W = 2\left[\frac{P_1 a}{RT} - \frac{P_2 a}{RT}\right] - b\left[P_1 - P_2\right]$$

$$= \frac{2a}{RT}(P_1 - P_2) - b(P_1 - P_2)$$

$$= (P_1 - P_2)\left[\frac{2a}{RT} - b\right]$$ → ④

If C_p be the specific heat at constant pressure then the amount of heat that must be supplied to restore the original temperature will be $-C_p dT$ calories. The value in joule will be $-C_p dT J$. Hence

$$-C_p dT J = (P_1 - P_2)\left[\frac{2a}{RT} - b\right]$$ → ⑤

Eq⑤ ⟹ $$dT = \frac{-(P_1 - P_2)}{C_p J}\left[\frac{2a}{RT} - b\right]$$ → ⑤

Here eq⑤ represents the expression for cooling produced

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and it shows that ① At any temperature T, the cooling produced is directly proportional to pressure diff $(P_1 - P_2)$ ② for a given pressure diff $(P_1 - P_2)$, the cooling is greater for lower initial temperature T.

Distinction between Joule's Expansion, Adiabatic Expansion and Joule - Kelvin Expansion:

① Joule's Expansion: It is a free expansion in which the gas expands into vacuum and does no external work. In this internal work is done by the gas against the intermolecular attraction.

The contribution of next external work may be a cooling effect or a heating effect, depending upon the initial temperature of the gas. This effect is not observed due to the large heat Capacity of the apparatus.

② Adiabatic Expansion: In this expansion the compressed gas is suddenly released to atmosphere. The gas performs external work against the pressure of the atmosphere.

The process is so fast that no appreciable heat flows into gas from the internal energy of the gas. The gas cools. So in adiabatic expansion the cooling is due to external work.

③ Joule - kelvin Expansion:- In this expansion the gas under high pressure is forced to pass through porous plug to a region of lower pressure. Here external work is done on gas as well as by the gas in addition to the internal work. There may be cooling effect or heating effect depending upo

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the initial temp of the gas. At ordinary temp gases show cooling effect.

Kapitza's Method:

out of syllabus

Kapitza's liquifier is shown in fig.

Pure and dry helium is compressed to 30 atmos by the Compressor P. Heat of Compression is removed by flowing water in a jacket surrounding a portion of the tube. The gas is then pre-cooled by passing it through a bath of liquid nitrogen boiling under reduced pressure.

The pre-cooled gas is divided into two parts at A in the heat exchanger. One part passes to the expansion engine E. Due to adiabatic expansion of the

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gas it cools to $-263^\circ C$. This cooled gas rises up in the heat exchanger and cools the second part coming towards the nozzle N. The second part is now so cooled that after the expansion at nozzle N, it is liquified. The liquified portion is collected in Dewar flask and the remaining part goes back to the compressor through heat exchanger.) out of syllabus

Imp

Adiabatic Demagnetisation :-

Principle :- When a paramagnetic substance already magnetised is suddenly demagnetised, it shows a slight fall in temperature. If the substance is already at a sufficient low temperature, the fall in temperature is considerable.

Method :- The apparatus is shown in fig. The paramagnetic substance (gadolinium-Sulphate) is suspended in vessel A connected to a vaccum pump. The vessel A is surrounded by liquid helium at the temperature 1k placed in Dewar flask B. This is again surrounded by another Dewar flask C containing liquid hydrogen. A strong magnetise the paramagnetic salt. The coils M round A are used to measure the temperature by susceptibility measurement.

Now the magnetising field is switched on. The paramagnetic salt becomes warmed but the heat flows out through the helium gas into the liquid helium. The temperature of P again falls to 1k. After this helium gas is now pumped out from the

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vessel A so that the paramagnetic substance becomes thermally insulated. The magnetic field is switched off. Due to adiabatic demagnetisation of substance its temperature falls.

Theory:

When a paramagnetic substance is magnetised its molecules try to align in the direction of field and work is done. Let the paramagnetic substance is placed the magnetic field of magnetic flux density B. Now the intensity of magnetisation per gram mole I of the substance is BdI. When the substance is demagnetised the work done by the substance is $-BdI$.

From first law of thermodynamics, we have

$$dQ = du + dw \qquad \{\because dw = Pdv - BdI\}$$
$$= du + Pdv - BdI$$

As the demagnetisation is carried out at constant pressure, there is no change in volume dv and hence $dv = 0$.

Now $dQ = du - BdI$

But $dQ = Tds$ where ds is the change in entropy

$$Tds = du - BdI \longrightarrow ①$$

The second Maxwells thermodynamic relation b/w entropy and pressure is expressed as

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

To obtain similar relation following eq ① we substitute $-B$ for P and I for V and get

$$\left(\frac{\partial T}{\partial B}\right)_S = -\left(\frac{\partial I}{\partial S}\right)_B$$

$$\left(\frac{\partial T}{\partial B}\right)_S = -\left[\frac{(\partial I/\partial T)_B}{(\partial S/\partial T)_B}\right] \longrightarrow ②$$

If m be the mass of specimen and C_B it specific heat at constant filed then

$$\left(\frac{\partial S}{\partial T}\right)_B = \frac{1}{T}\left(\frac{\partial Q}{\partial T}\right) = \frac{mC_B}{T}$$

Subtitute this value in eq ② we have

$$\left(\frac{\partial T}{\partial B}\right)_S = \frac{-T}{mC_B}\left(\frac{\partial I}{\partial T}\right)_B$$

$$\partial T = \frac{-T}{mC_B}\left(\frac{\partial I}{\partial T}\right)_B \partial B \rightarrow \textcircled{3}$$

Eq ③ gives the fall in temperature of the specimen due to adiabatic demagnetisation.

The susceptibility χ of the substance is defined as

$$\chi = \frac{\text{Intensity of magnetisation of specimen}}{\text{Intensity of magnetizing field}} = \frac{I}{B} \rightarrow \textcircled{4}$$

According to curies law, the paramagnetic susceptibility χ of a substance is inversly proportional to the absolute temperature T

$$\chi \propto \frac{1}{T} \text{ (or) } \chi = \frac{C}{T} \rightarrow \textcircled{5}$$

where C is constant

from eq ④ and ⑤

$$\frac{I}{B} = \frac{C}{T} \text{ (or) } I = \frac{CB}{T} \rightarrow \textcircled{6}$$

Substitute the value eq I from eq ⑥ in eq ③ we get

$$\partial T = \frac{-T}{mC_B}\left\{\frac{\partial}{\partial T}\left(\frac{CB}{T}\right)\right\} dB \rightarrow \textcircled{7}$$

Let T_i and T_f be the initial and final temperature of the specimen corresponding to the initial and final magnetic flux densities B_i and B_f, then

$$T_f - T_i = \frac{T}{mC_B}\int_{B_i}^{B_f}\left\{\frac{\partial}{\partial T}\left(\frac{C_B}{T}\right)\right\}\partial B$$

Simplifying, we get.

$$T_f - T_i = \frac{C}{2mC_BT}\left(B_f^2 - B_i^2\right)$$

$$= \frac{k}{2C_BT}\left(B_f^2 - B_i^2\right)$$

where $k = (C/m)$ called as curies constant per unit meter when field is switched off then $B_f = 0$, Hence

$$T_f - T_i = \frac{-kB_i^2}{2C_BT} \rightarrow ⑧$$

The -ve sign in eq ⑧ indicates a fall in temperature in the adiabatic demagnetisation.

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(This is done because carbon-dioxide and water vapour will solidify before the air is liquified and choke the whole system.

Air free from carbon-dioxide and water vapour now enters the second compressor where air then pas

atmospheric pressure. The compressed air then passes through the spiral tube kept in freezing mixture. The temperature of air here falls to $-20°$. The air thus, cooled at high pressure enters the liquifying section and suffers Joule - Thomison expansion of nozzle N_L.

matter when

As the gas at this stage is allowed to expand to atmos pressure, its temperature falls to about $-70°C$. The nozzle N_2 is closed at this stage. So the cooled air flows through the wider tube B back to the compressor P_2, where it is again compressed to 200 atmosphere pressure.

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out of syllabus

In its way to compression P_2 it cools the incoming gas in tube A. The principle of regenerative cooling takes place and after few cycles the air is cooled to sufficiently low temperature ($-183°C$) Now the second nozzle N_2 is opened. The air expands to one atmosphere pressure and is liquified. The liquid air is collected in Dewar flask D.]

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Properties of substance at low temperatures:-

At low temperature the matter exhibits same peculiar properties as compared to the properties shown at ordinary temperature. At low temperatures and very low temperatures the following are the few properties of the substances.

① At temperature below 90 k, most of the chemical reactions causes to take place. At these temperatures animal and vegetable matter can be ~~pressured~~ preserved without putrification.

2) Certain materials like cotton and wool exhibit property of flurosecence when exposed to the temperature of liquid air for source time.

3) At temperatures below 83k lead loses elasticity and attains a plastic state. (Rubber into pieces like cork).

4) For all substances the atomic heat tends to become zero as the absolute zero of temperatures is reached.

5) The magnetic susceptibility of certain paramagnetic salts like copper sulphate vary inversely as the absolute temperature in the region of low temperature.

6) The entropy of the substances tends to zero value when it is exposed to very low temperature.

7) The electrical resistance is reduced at a particular temperature is called transition temperature is 4.2k. The property of a

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Joule Expansion	Adiabatic Expansion	Joule - Kelvin Expansion
1) It is free expansion the gas expands in vaccum itself	1) Compressed gas is suddenly released into atmosphere which expands.	1) A highly compressed gas is forced to pass through a porous plug into a low pressure region.
2) No external work is done by the gas	2) The gas performs external work against pressure of the atmosphere	2) In this case external work is done on the gas as well as by the gas in addition to internal work.
3) The internal work is done by the gas against inter-molecular attraction	3) In this expansion no heat transfer takes place	3) There may be cooling effect or heating effect depending on the gas initial temp of gas.
4) There may be a heating effect or a cooling effect depending upon the initial temp of the gas.	4) The external work is drawn from the internal energy of the gas and therefore the gas cools.	4) Cooling is when the temp of the gas is less than inversion temp and above the inversion temp there will be a heating effect.

3. Thermodynamic potentials and Maxwell's equations ①

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Thermodynamic potentials :-

Thermodynamic potentials are the energy functions that are formed by combining the basic thermodynamic co-ordinates P, V, T and S. As the another four quantities U, H, F, and G also denote the state of system in thermodynamics. and are called thermodynamic potentials

① Internal Energy U :-

According to 1st Law of thermodynamics the internal energy of the system U is a State variable and is a point function.

When the system passes from one equilibrium state to another the change in internal energy ΔU doesnot depend upon the path followed, but depends only on the initial and final State. In a complete cycle the change in internal energy is zero.

According to 1st Law

$$dU = dQ - dW \longrightarrow ①$$

at Const pressure $\delta W = PdV$

From second Law $\delta Q = TdS$

Then finally $\boxed{dU = TdS - PdV} \longrightarrow ②$

② The enthalpy H :-

The enthalpy H is an energy function defined by the relation

$$\boxed{H = U + PV} \longrightarrow ③$$

This State function H gives us the heat content at constant pressure. The change in enthalpy is equal to the quantity of heat given to the System at constant pressure.

differentiate eq ③ $dH = dU + PdV + VdP$

$$= (TdS - \cancel{PdV}) + \cancel{PdV} + VdP$$

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$$\boxed{dH = TdS + VdP} \longrightarrow ④$$

One of the most interesting properties of the enthal, and function is in connection with throttling process, this process H is const.

iso
funct
as
Pt

③ The Helmholtz free energy F :-

The Helmholtz function F is defined by the relation

$$\boxed{F = U - TS} \longrightarrow ⑤$$

diff eq ⑤ we get

$$dF = dU - TdS - SdT$$

we have $TdS = dU + PdV$

Then $dF = dU - dU - PdV - SdT$

$$= -PdV - SdT$$

$$\therefore \boxed{dF = -PdV - SdT} \longrightarrow ⑥$$

In an isothermal - isochoric process only those states are stable for which the Helmholtz free energy is a minimum. From eq ⑤ as the entropy S is increased, the available energy F gets decreased. F is called the available energy at const temp.

④ The Gibb's free energy or Gibb's function :-

The Gibb's function G is defined by the relation

$$\boxed{G = H - TS} \longrightarrow ⑦$$

diff eq ⑦ we get

$$dG = dH - TdS - SdT$$

But $dH = TdS + VdP$

$$\therefore dG = (TdS + VdP) - TdS - SdT$$

$$= (VdP - SdT)$$

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$$\boxed{dG = -SdT + VdP} \longrightarrow ⑧$$

In the case of a reversible isothermal (dT=0) and isobaric (dP=0) process dG=0 and G = Const. Gibbs function is of very important significance in chemistry as chemical reactions occur at const temp and const pressure. Gibbs function denotes the available energy at a const pressure. G is also called thermal potential.

They are of practical importance in studying the conditions of equilibrium of a system.

→ Maxwell's Equations :-

Maxwell equations are the differential relations among the four basic thermodynamic co-ordinates P, V, T and S. These relations are derived from the four thermodynamic potentials U, H, F, and G.

The importance of Maxwell's equations lies in the fact that these equations provide us with relationships b/w measurable quantities and quantities that cannot be measured.

Mathematical theorem :-

If there exists a relationship among x, y and z then we can express z as a function of x and y. And hence we can write.

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy.$$

If we let $\left(\frac{\partial z}{\partial x}\right)_y = M$ and $\left(\frac{\partial z}{\partial y}\right)_x = N$

Then $dz = Mdx + Ndy$.

where z, M, N are all functions of x and y. partial differentiation of M with respect to y and N with respect to x yeilds us

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x \partial y} \quad \text{and}$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x \partial y}$$

and $$\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y \partial x}$$

As the right hand side terms are equal in the two equations

$$\boxed{\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y .} \longrightarrow ①$$

This is the Condition for an exact differential.

① Let us take up the equation

$\boxed{du = TdS - PdV}$ and compare with

$dz = Mdx + Ndy$

This leads to $M \longrightarrow T$
$x \longrightarrow S$
$N \longrightarrow -P$
$y \longrightarrow V$

from eq ① we get

$$\boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V} \longrightarrow ②$$

This is Maxwell's first equation.

② Let us take up the equation

$dH = TdS + VdP$ and compare with
$dz = Mdx + Ndy$

This leads to
$M = T$
$x = S$
$N = V$
$y = P$

from eq ① $$\boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P} \longrightarrow ③$$

This is Maxwell's Second equation

) Let us take up the equation

$$dF = -SdT - PdV \quad \text{and Compare with}$$
$$dz = Mdx + Ndy$$

This leads to
$M = -S$
$x = T$
$N = -P$
$y = V$

Now from eq ① we get

$$\boxed{\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V} \longrightarrow ④$$

This is Maxwell's ~~eq~~ third equation

④ Let us take up the equation

$$dG = -SdT + VdP \quad \text{and compare with}$$
$$dz = Mdx + Ndy$$

This leads us to
$M = -S$
$x = T$
$N = V$
$y = P$

Now from eq ① we get

$$\boxed{\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P} \longrightarrow ⑤$$

This is Maxwell's fourth equation.

→ Ratio of the two Specific Heats C_p and C_v:-

The coefficient of volume elasticity is defined as

$$E = \frac{\text{Stress}}{\text{volume Strain}} = \frac{dP}{-\left(\frac{dV}{V}\right)} = -V\left(\frac{dP}{dV}\right)$$

The negative sign indicating that the volume decreases as pressure increase.

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The adiabatic elasticity E_S (at const entropy) $= -V\left(\frac{\partial P}{\partial V}\right)_S$

and the isothermal elasticity E_T (at const temp) $= -V\left(\frac{\partial P}{\partial V}\right)_T$

$$\therefore \frac{E_S}{E_T} = \frac{\left(\frac{\partial P}{\partial V}\right)_S}{\left(\frac{\partial P}{\partial V}\right)_T} = \frac{\left[\left(\frac{\partial P}{\partial T}\right)\left(\frac{\partial T}{\partial V}\right)\right]_S}{\left[\left(\frac{\partial P}{\partial S}\right)\left(\frac{\partial S}{\partial V}\right)\right]_T}$$

$$= \frac{{}^{*}\left(\frac{\partial P}{\partial T}\right)_S\left(\frac{\partial T}{\partial V}\right)_S}{{}^{*}\left(\frac{\partial P}{\partial S}\right)_T\left(\frac{\partial S}{\partial V}\right)_T}$$

But from Maxwell's equations

$$\left(\frac{\partial P}{\partial T}\right)_S = \frac{1}{\left(\frac{\partial T}{\partial P}\right)_S} = \frac{1}{\left(\frac{\partial V}{\partial S}\right)_P} = \left(\frac{\partial S}{\partial V}\right)_P$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial P}{\partial S}\right)_T = \frac{1}{\left(\frac{\partial S}{\partial P}\right)_T} = \frac{1}{-\left(\frac{\partial V}{\partial T}\right)_P} = -\left(\frac{\partial T}{\partial V}\right)_P$$

and $$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Therefore

$$\frac{E_S}{E_T} = \frac{\left(\frac{\partial S}{\partial V}\right)_P\left(\frac{\partial P}{\partial S}\right)_V}{\left(\frac{\partial T}{\partial V}\right)_P\left(\frac{\partial P}{\partial T}\right)_V}$$

$$\left[\frac{\left(\frac{\partial s}{\partial V}\right)}{\left(\frac{\partial T}{\partial V}\right)}\right]_p \cdot \left[\frac{\left(\frac{\partial p}{\partial s}\right)}{\left(\frac{\partial p}{\partial T}\right)}\right]_V = \left(\frac{\partial s}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial s}\right)_V$$

$$= \frac{\left(\frac{\partial s}{\partial T}\right)_p}{\left(\frac{\partial s}{\partial T}\right)_V}.$$

But from Q Clausius equality $\partial Q = T ds$ we have

$$T\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial Q}{\partial T}\right)_p = C_p$$ and

$$T\left(\frac{\partial s}{\partial T}\right)_V = \left(\frac{\partial Q}{\partial T}\right)_V = C_V$$

$$\therefore \frac{E_s}{E_T} = \frac{\left(\frac{\partial s}{\partial T}\right)_p}{\left(\frac{\partial s}{\partial T}\right)_V} = \frac{C_p}{C_V}$$

The ratio of adiabatic and isothermal elasticities is a const γ.

Thus the ratio specific heats of substance is equal to the ratio of its adiabatic and isothermal elasticities

$$\boxed{\frac{C_p}{C_V} = \frac{E_s}{E_T} = \gamma = \frac{K}{K_s}}$$

Where $K_s = \frac{1}{E_s}$ is the adiabatic compressibility and $K = \frac{1}{K}$ is the isothermal compressibility.

→ **Difference of the two Specific heats cpa::**

C_p

for one

The specific heat at const pressure is defined

$$c_p = \left(\frac{\partial Q}{\partial T}\right)_p$$

and the Specifice heat at const volume is defined by

$$c_V = \left(\frac{\partial Q}{\partial T}\right)_V$$

$$\therefore c_p - c_V = \left(\frac{\partial Q}{\partial T}\right)_p - \left(\frac{\partial Q}{\partial T}\right)_V$$

From clausius equality $\delta Q = Tds$ and we have

$$\left(\frac{\partial Q}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p \text{ and } \left(\frac{\partial Q}{\partial T}\right)_V = T\left(\frac{\partial s}{\partial T}\right)_V$$

$$\therefore c_p - c_V = T\left(\frac{\partial s}{\partial T}\right)_p - T\left(\frac{\partial s}{\partial T}\right)_V = T\left[\left(\frac{\partial s}{\partial T}\right)_p - \left(\frac{\partial s}{\partial T}\right)_V\right]$$

Now let us consider entropy s of a function of V and T

$$s = s(V,T)$$

we can write $ds = \left(\frac{\partial s}{\partial T}\right)_V dT + \left(\frac{\partial s}{\partial V}\right)_T dV$

devide with dT

$$\therefore \left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial s}{\partial T}\right)_V + \left(\frac{\partial s}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

$$\& \left(\frac{\partial s}{\partial T}\right)_p - \left(\frac{\partial s}{\partial T}\right)_V = \left(\frac{\partial s}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_p$$

$$= \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$$

From Maxwells 3^{rd} equations $\left(\frac{\partial s}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

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pa

defined

$$\therefore \boxed{C_p - C_V = T \cdot \left(\frac{\partial P}{\partial T}\right)_V \cdot \left(\frac{\partial V}{\partial T}\right)_P} \longrightarrow ①$$

for one mole of an ideal gas, we have the gas eq

$$PV = RT$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V} \quad \text{and} \quad \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$$

Hence $C_p - C_V = T \cdot \frac{R}{V} \cdot \frac{R}{P} = \frac{T \cdot R \cdot R}{P \cdot V} = \frac{\cancel{T}RR}{\cancel{RT}} = R$.

That is $\boxed{C_p - C_V = R}$ for one mole of an ideal gas.

→ Clausius Clapeyron's equation:-

The clausius clapeyron latent heat equation connects the change in melting point & boiling point with the change in pressure. This can be derived from Maxwell's equations as follows.

Let

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \longrightarrow ① \quad \text{Max } 3^{rd} \text{ Law}$$

Multiply ~~to~~ with T on both sides

$$T\left(\frac{\partial P}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial Q}{\partial V}\right)_T$$

as T is const and from $\partial Q = T ds$ [clausius equation

Here ∂Q ~~p~~ represents the heat absorbed at const temp. When heat is absorbed at const temp when the system undergoes a change of phase, Then heat is called the latent Heat.

Let the latent heat absorbed during phase change be L. Let the specific volumes of substance be V_i and V_f in the first state and second respectively. Then $\partial V = V_f - V_i$ and the above equation becomes

$$T\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{L}{V_f - V_i}\right)_T \quad \text{or}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{L}{T(V_f - V_i)}$$

Finally

$$\boxed{\frac{\partial P}{\partial T} = \frac{L}{T(V_f - V_i)}} \longrightarrow ②$$

This equation is called clausius clapeyron's Latent Heat equation.

This equation refers to first order phase transitions which are by ~~definitio~~ the emission & absorption of heat with out any change in temperature.
Ex:- melting, vaporization and sublimation

Application :-

① used to study the variation of the boiling point of a liquid with pressure

② used To study variation of ~~melti~~ melting point of a solid with pressure.

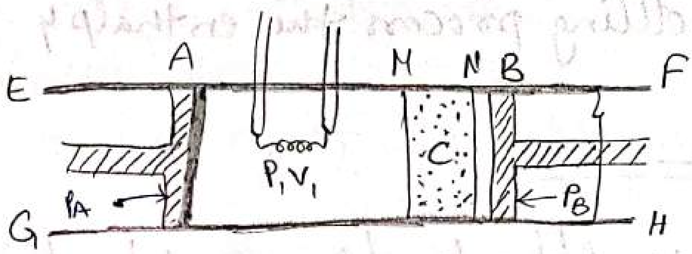
③ It is used to explain regelation of ice :- when two pieces of ice ~~a~~ are pressed together they form into a single piece.

→ Throttling / process :- Joule-Kelvin effect :-

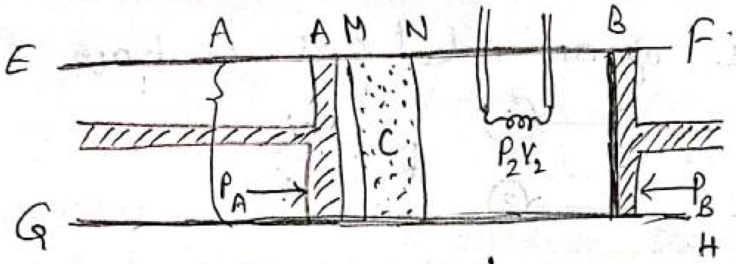
when a gas under a const high pressure is passed through a porous plug (cotton) to a region of constant low pressure, there will be a change in its temp. This was first observed by Joule and Kelvin and hence is called the Joule-Kelvin (Joule Thomson) effect. The process of passing of the gas under such conditions is called throttling process.

Let as consider a cylinder EFGH that is thermally insulated from the surroundings. The cylinder has got two non conducting pistons P_A and P_B that can be moved inside the cylinder. Firstly P_B is just to the right of porous plug C contained in the region MN and the piston P_A is at A to the left of porous plug at a distance away from it.

B/W P and C there is a gas at P_1 and V_1. At initial state gas is an equilibrium state. Now let us move both pistons P_A and P_B then high pressure P_1 is maintained on the left side of the porous plug and a const low pressure P_2 is maintained on the right hand side of the porus plug.



ⓐ



ⓑ

After all gas (left of c) has seeped through the porous plug (now right of c) the final equilibrium state will be a shown in fig ⓑ. This is called throttling process.

This throttling process is an irreversible adiabatic process. from the first Law of thermodynamics

$$Q = (U_2 - U_1) + W \longrightarrow ①$$

As no heat enters & leaves the system

$$Q = 0 \longrightarrow ②$$

and $$W = \int_0^{V_2} p\,dV + \int_{V_1}^{0} p\,dV \longrightarrow ③$$

As we have ensured that P_1 and P_2 are maint… at Const values $W = P_2V_2 - P_1V_1 \longrightarrow ④$

This is the amount of work required to keep gas through the porus plug.

$$\therefore 0 = (U_2 - U_1) + P_2V_2 - P_1V_1 \longrightarrow ⑤$$

$$U_1 + P_1V_1 = U_2 + P_2V_2 \longrightarrow ⑥$$

i-e $\boxed{U + PV = \text{Const}}$ and $\boxed{U + PV = H}$, the en…

& $\boxed{H_i = H_f}$. (in a throttling process) $\longrightarrow ⑦$

Thus in a throttling process the enthalpy remain a Const.

Joule - Kelvin effect for an ideal Gas:-

For one mole of an ideal gas we have

$$PV = RT \longrightarrow ①$$

$$R = \frac{PV}{T}$$

$$V = \frac{RT}{P} \longrightarrow ②$$

$$\therefore \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P} \longrightarrow ③$$

and hence $T\left(\frac{\partial V}{\partial T}\right)_P = \frac{T.R}{P} = \frac{T}{P}.\left(\frac{PV}{T}\right) = V \longrightarrow ④$

$$\therefore T\left(\frac{\partial V}{\partial T}\right)_P - V = V - V = 0 \longrightarrow ⑤$$

Therefore $\mu = 0$ for an idealgas.

Thus the Joule Kelvin for an ideal gas is zero. The ideal gas undergoing a Joule-Kelvin expansion (a throttling process) will get neighter cooled nor heated.

Keep

Expression of Joule-Thomson Coefficient:-

Joule-Thomson observed that there is a change in temp of gas from above experiment. Hence this throttling process is also called the Joule-thomson expansion. As shown in throttling process, enthalpy of the system remains a const.

By def the enthalpy

$$H = U + PV \longrightarrow \textcircled{1}$$

$$dH = Tds + Vdp \longrightarrow \textcircled{2}$$

Second Tds eq, we have

$$\star \boxed{Tds = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp} \longrightarrow \textcircled{3}$$

$$\therefore dH = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp + Vdp \longrightarrow \textcircled{4}$$

$$= C_p dT - \left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] dp.$$

or $$C_p dT = dH + \left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] dp \longrightarrow \textcircled{5}$$

~~But dH = 0 in throttling process~~

But dH = 0 in throttling process.

$$\therefore dT = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] dp. \longrightarrow \textcircled{6}$$

But $T = T(P, H)$ $$dT = \left(\frac{\partial T}{\partial P}\right)_H dp + \left(\frac{\partial T}{\partial H}\right)_p dH \longrightarrow \textcircled{7}$$

But dH = 0 in throttling process.

$$\therefore \left(\frac{\partial T}{\partial P}\right)_H dp = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right) dp \longrightarrow \textcircled{8}$$

$$\& \boxed{\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]} \longrightarrow \textcircled{9}$$

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The Ratio of change of temp of a gas to the cha
of pressure during a throttling process at const
is defined as the Joule - Kelving coefficient an
is denoted by μ.

$$\therefore \quad \mu = \left(\frac{\partial T}{\partial p}\right)_H \longrightarrow (10)$$

Hence
$$\mu = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] \longrightarrow (11)$$

With
$$\mu = \left(\frac{\partial T}{\partial p}\right)_H$$

From definition of μ it is clear that from eq

① If μ is −ve, $\left(\frac{\partial T}{\partial p}\right)_H$ is −ve
Hence temp increases, pressure decreases.
Heating effect is produced.

② If μ is +ve, $\left(\frac{\partial T}{\partial p}\right)_H$ is +ve, Hence
temp ↓, pressure ↓
cooling effect produced.

③ If μ is zero $\left(\frac{\partial T}{\partial p}\right)_H$ is $= 0$. Hence there will be neither cooling nor heating. Temp at which the ~~colli~~ cooling changes into heat is called the Inversion Temp (Phenomena - Liquification of gases)

Const

cient are

Joule - Kelvin effect for a real gas obeying Vander waal's equation :-

We have the Vander waal's equation of state for a real gas as $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ (for one mole) $\longrightarrow$ ①

Keeping P as a const, we can have the partial derivative of V w.r.t T as

$$\left(P+\frac{a}{V^2}\right)\left(\frac{\partial V}{\partial T}\right)_P+(V-b)\times\left[-\left(\frac{2a}{V^3}\right)\left(\frac{\partial V}{\partial T}\right)_P\right]=R \longrightarrow ②$$

$$\left(\frac{\partial V}{\partial T}\right)_P=\frac{R}{\left(P+\frac{a}{V^2}\right)-\frac{2a}{V^3}(V-b)} \longrightarrow ③$$

$$=\frac{R}{\left(\frac{RT}{V-b}\right)-\frac{2a(V-b)}{V^3}}=\frac{R(V-b)}{RT-\frac{2a(V-b)^2}{V^3}}$$

$$=\frac{R(V-b)}{\left(RT-\frac{2a}{V}\right)} \longrightarrow ④ \rightarrow$$ Multiply with T on both sides and substract with V

$$\therefore T\left(\frac{\partial V}{\partial T}\right)_P-V=\frac{RT(V-b)}{RT-\frac{2a}{V}}-V \longrightarrow ⑤$$

$$=\frac{RT(V-b)-RTV+2a}{\left(RT-\frac{2a}{V}\right)}$$

$$=\frac{2a-RTb}{RT\left(1-\frac{2a}{RTV}\right)}=\left(\frac{2a-RTb}{RT}\right)\left(1-\frac{2a}{RTV}\right)^{-1} \longrightarrow ⑥$$

$$=\left[\frac{2a}{RT}-b\right]$$ neglecting $\frac{2a}{RTV}$ w.r.t 1

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Finally the Joule-Kelvin Coefficient is given by the equation

$$\mu = \left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] \longrightarrow ⑦$$

$$\therefore \boxed{\mu = \frac{1}{C_p}\left[\frac{2a}{RT} - b\right]} \longrightarrow ⑧$$

It is evident from this equation that if $\frac{2a}{RT} = b$, then $\mu = 0$ and $\left(\frac{\partial T}{\partial P}\right)_H = 0$ and hence there will be neither cooling nor heating of the gas That is the inversion temp is defined by the eq

$$\frac{2a}{RT_i} = b \quad \& \quad T_i = \frac{2a}{Rb} \longrightarrow ⑦$$

For a real gas obeying vander waal's eq, there will be cooling during a Joule-Kelvin expansion When $T < T_i$ i.e $\boxed{T < \frac{2a}{Rb}}$

then μ is +ve, and heating when $T > T_i$, $T > \frac{2a}{Rb}$ and μ is -ve.

→ J-K effect - perfect gas:-

For one mole of an ideal gas

$PV = RT \Rightarrow R = \frac{PV}{T}$

$\left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{P}$

$T\left(\frac{\partial V}{\partial T}\right)_p = T\frac{R}{P} = \frac{\cancel{T}}{\cancel{P}}\cdot\left(\frac{\cancel{P}V}{\cancel{T}}\right) = V$

$T\left(\frac{\partial V}{\partial T}\right)_p - V \Rightarrow V - V = 0$

Therefore $\mu = 0$ for ideal gas Thus the Joule Kelvin for an ideal gas is zero. The ideal gas undergoing a Joule-Kelvin expansion (a throttling process) will get neither cooled nor heated.

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I 1. Kinetic Theory of gases

Introduction :- The kinetic theory of an ideal gas deals with the molecules of the gas, their internal motion as well as their external motion, their collisions among themselves and their collisions with the walls of the container and the forces of interaction b/w molecules.

The Laws of mechanics and the theory of probability are the tools in kinetic theory of gases.

[redacted]

→ Maxwell's Law of Distribution of Molecular Speeds :-

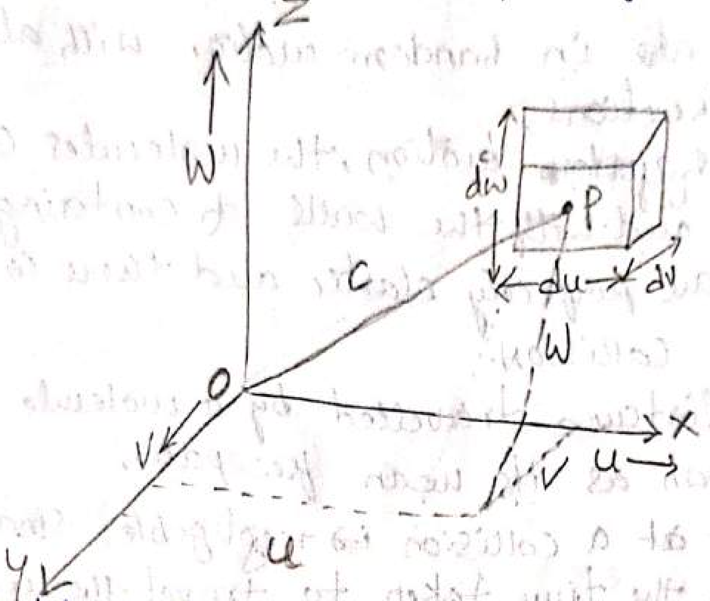
According to kinetic theory of gases, a gas consists of a large no of molecules and are frequently colliding with other molecule as well as with the walls of the containing vessel, so their speeds and directions of motion are changing. At a given temp the root mean square speed of the gas molecules remains unchanged.

The Law of distribution of speeds among the molecules of the gas was first established by Maxwell. The proof of the Law was given by Boltzmann and this Law is known as Maxwell's Boltzmann's distribution Law for molecular speed.

According to Maxwell's Law of distribution of molecular speeds, the no. of gas molecules dn_c having their Speeds b/w c and c+dc.

$$\boxed{dN_c = 4\pi n \left(\frac{m}{2\pi kT}\right)^{3/2} \bar{e}^{\,mc^2/2kT} c^2\, dc}$$

where n = total no of gas molecules
m = mass of each gas molecule.
k = Boltzmann Const
T = absolute temp. of gas.



Now let us represent the molecules in a velocity diagram with O as the origin and OX, OY and OZ are co-ordinate axes as shown in fig.

Now the molecule having velocity c represented by OP = c (Vec

image ← The projections in x, y, z planes are u, v, w and velocity components are u, u+du, v, v+dv, w, w+dw will be contained in the element of volume dudvdw is a function of c and dudvdw.

This may be expressed as f(c) dudvdw (or)

$\phi(c^2)$ dudvdw.

So we can write

$$f(u)f(v)f(w)\,du\,dv\,dw = \phi(c^2)\,du\,dv\,dw$$

$$f(u)f(v)f(w) = \phi(c^2) = \phi(u^2+v^2+w^2) \xrightarrow{\quad} ①$$

where $c^2 = u^2+v^2+w^2$.

To solve eq ① chose $\phi(c^2)$ = Const

differentiate we get $d[\phi(c^r)]=0 \longrightarrow ②$

Differentiating eq ①, we get

$$d[f(u)f(v)f(w)] = d[\phi(c^r)] = 0$$

$$f'(u)f(v)f(w)\,du + f(u)f'(v)f'(w)\,dv + f(u)f(v)f'(w)\,dw = 0 \longrightarrow ③$$

def. devide eq ③ by $f(u)f(v)f(w)$ we have

$$\frac{f'(u)}{f(u)}du + \frac{f'(v)}{f(v)}dv + \frac{f'(w)}{f(w)}dw = 0 \longrightarrow ④$$

also we can write $c^r = u^r + v^r + w^r$ ——

Differentiating we get

$$0 = 2u\,du + 2v\,dv + 2w\,dw$$

$$0 = u\,du + v\,dv + w\,dw \longrightarrow ⑤$$

Multiply eq ⑤ with λ we get

$$0 = \lambda u\,du + \lambda v\,dv + \lambda w\,dw \longrightarrow ⑥$$

where λ is constant quantity.

add eq ④ + ⑥

$$\left[\frac{f'(u)}{f(u)} + \lambda u\right]du + \left[\frac{f'(v)}{f(v)} + \lambda v\right]dv + \left[\frac{f'(w)}{f(w)} + \lambda w\right]dw = 0 \longrightarrow ⑦$$

since u, v, w are independent of each other, eq ⑦ can be satisfied only if each of the terms is sperately equal to zero. Therefore

$$\left[\frac{f'(u)}{f(u)} + \lambda u\right]du = 0 \; ; \; \left[\frac{f'(v)}{f(v)} + \lambda v\right]dv = 0 \; ; \; \left[\frac{f'(w)}{f(w)} + \lambda w\right]dw = 0$$

$$\longrightarrow ⑧$$

Now consider the 1st equation eq ⑧ we get

$$\frac{f'(u)}{f(u)} du = -\lambda u du.$$

Integrating

$$\boxed{\log_e (fu) = -\lambda \frac{u^2}{2} + \log_e a}$$

where $\log_e a$ is a const of integration

$$\log_e\left[\frac{f(u)}{a}\right] = \frac{-\lambda u^2}{2}.$$

or $\left[\frac{f(u)}{a}\right] = e^{-\lambda u^2/2}$ or $f(u) = a e^{-\lambda u^2/2} = a e^{-bu^2}$ $\xrightarrow{\quad}$ ⑨ where $\lambda/2 = b$.

$f(u) = a e^{-bu^2} \longrightarrow$ ⑩

Similarly $f(v) = a e^{-bv^2} \longrightarrow$ ⑪

$f(w) = a e^{-bw^2} \longrightarrow$ ⑫

from eq ⑨, ⑩, ⑪

$$f(u) f(v) f(w) = a^3 e^{-b(u^2+v^2+w^2)} \longrightarrow ⑬$$

The const a, b can be found by integrating the above eq applying the boundary conditions. Let n be the no of molecules per c c of the gas with all possible velocities from $-\infty$ to $+\infty$. According to the definition of probability

$$n \iiint_{-\infty}^{+\infty} f(u) f(v) f(w)\, du\, dv\, dw = n.$$

$$\iiint_{-\infty}^{+\infty} f(u) f(v) f(w)\, du\, dv\, dw = 1$$

or $$\iiint_{-\infty}^{+\infty} a^3 e^{-b(u^2+v^2+w^2)} du\, dv\, dw = 1.$$

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we know that the value of definite integral

$$\iiint_{-\infty}^{+\infty} e^{-bu^2}\, du = \sqrt{\frac{\pi}{b}}.$$

$$a^3\left(\sqrt{\frac{\pi}{b}}\right)\left(\sqrt{\frac{\pi}{b}}\right)\left(\sqrt{\frac{\pi}{b}}\right) = 1$$

or $a^3\left(\frac{\pi}{b}\right)^{3/2} = 1$ & $\boxed{a = \left(\frac{b}{\pi}\right)^{1/2}} \longrightarrow$ ⑭

we know that $b = \frac{m}{2KT}$ where m - mass
K - Boltzman const
T - absolute Temp

Thus $a = \left(\frac{b}{\pi}\right)^{1/2} = \left(\frac{m}{2\pi KT}\right)^{1/2} \longrightarrow$ ⑮

and $b = \frac{m}{2KT}$

Thus the no. of dn of molecules having velocity components lying b/w u and u+du, v and v+dv, w and w+dw is given by

$$dn = n f(u) f(v) f(w)\, du\, dv\, dw$$

Substitute the value of $f(u)f(v)f(w)$ from eq ⑬

$$dn = n a^3 e^{-b(u^2+v^2+w^2)}\, du\, dv\, dw.$$

Again substitute the values of a and b from eq ⑬

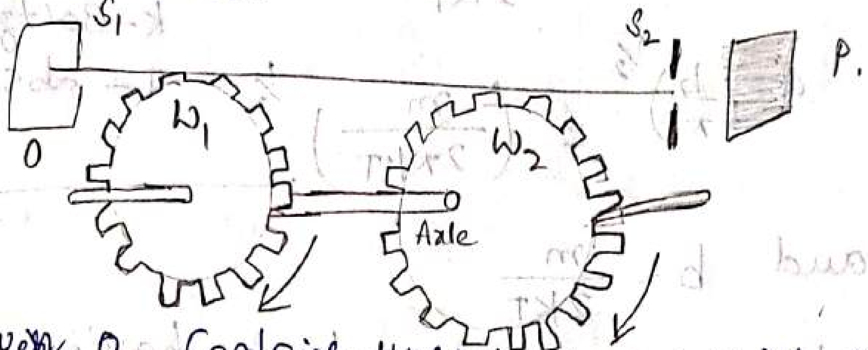
$$dn = n\left(\frac{m}{2\pi KT}\right)^{3/2}\left(e^{-m/2KT}\right)\left[u^2+v^2+w^2\right] du\, dv\, dw \longrightarrow ⑯$$

This is Maxwell's distribution Law.

→ Experimental verification of Toothed Wheel Experiment:-

Lammert's Toothed wheel Method

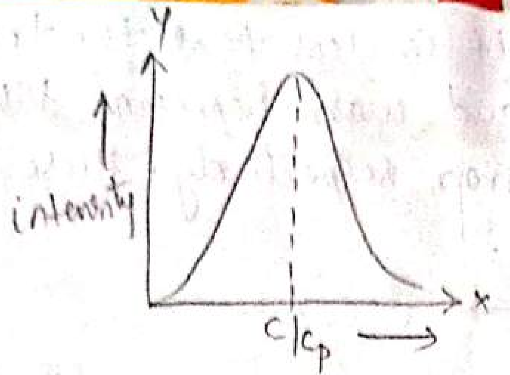
Lammert in 1926 used the toothed wheel method for sorting out molecules having different velocities. The experimental arrangement is shown in fig. Two equal toothed wheels W_1 and W_2 (50 teeth in each) are mounted on the same axle. The slits in W_2 are not parallel to those in W_1 but are displaced at an angle of 2° behind them.



An oven O contains mercury vapours which comes out of a slit S_1 with the help of another slit a thin and parallel molecular beam is obtained. The wheels are set in rapid rotation. Mercury is then heated in the oven, and the beam ~~passing~~ passing through slit S_1 falls on the slot W_1 now fall on W_2.

The principle of the experiment is based on the ~~the~~ fact that those molecules are able to pass through a slot in W_2 which have the requisite speed to cover the distance W_1W_2 in the small interval of time taken by the wheel to trun through an angle of 2°.

By rotating the wheel with various speeds, the molecules of different speeds are collected at different places on the plate P. The relative intensities of these collections are measured by microphotometer. From this measurement, the relative no of molecules lying in different ranges are calculated.



The intensity of spot formed on p is plotted against the ratio c/c_p of the molecules at a given temp and the graph is shown above. It was observed from the graph that it resembels well with the theoritical curve of Maxwell's distribution law of molecular speeds.

→ Transport phenomena :-

We have applied kinetic theory of gases when the gas is in equilibrium state. But if the gas is not in equilibrium state we have the following 3 cases

① The components of velocity (u, v, w) may not have same value in all parts of the gas, this will result in relative motion of the gas layers w.r.to one another. Now the layer moving faster impart & transport momentum to the slower moving layers to bring equilibrium state. This gives rise to the phenomena of viscosity.

② The different parts of the gas may be at different temps. Now to bring the equilibrium state, the molecules of the gas will transport K.E from regions of higher temp to region of low temp. This gives rise to the phenomena of conduction

③ The different parts of gas may have different densities i.e no of molecules per unit volume. Now to bring the equilibrium state the molecules of the gas trasport mass from region of higher density to region of lower density. This gives rise to the phenomenon of diffusion.

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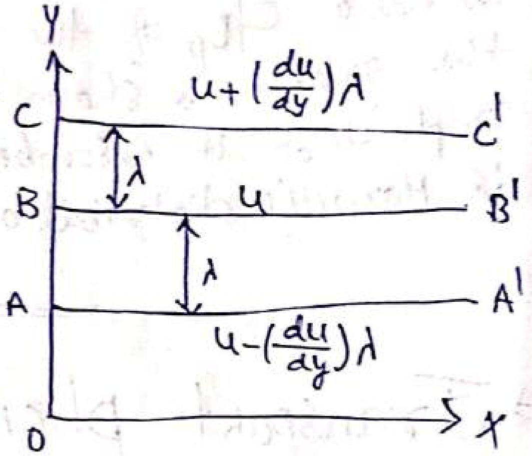
From above discussion, it is clear that the transport of momentum, energy and mass represent viscosity, conduction and diffusion respectively. These are called transport phenomena.

→ Viscosity of Gases :-



Let ~~us~~ the gas be moving in the +ve x-direction on solid horizontal surface OX as shown in fig. For a liquid contact with surface the flow velocity of the gas is zero and increase gradually along OY direction, and is said to be the gas is moving under steady state flow.

The gas at steady state flow may be considered that there are many layers parallel to fixed surface OX. At the ~~ce~~ surface the velocity is zero. As the flow is a steady state flow we can assume a constant velocity gradient $\frac{du}{dy}$ b/w different layers. If the flow velocity of ~~the~~ gas in layer BB' is 'u' then the flow velocity of the gas in a layer CC' is just at a distance λ above B will be equal to

$$u+\left(\frac{du}{dy}\right)\lambda \longrightarrow ①$$

Velocity of gas along the layer AA' is $u-\left(\frac{du}{dy}\right)\lambda$ (below BB') $\longrightarrow ②$

The coefficient of viscosity of a fluid is defined as the tangential force per unit area per unit velocity ~~grain~~ gradient b/w the layers of the fluid.

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Newton showed that this viscous force is directly proportional to the surface area A and the velocity gradient $\frac{du}{dy}$.

That is $F = -\eta A \frac{du}{dy} \longrightarrow$ ③

The -ve sign indicates that the viscous force decreases as we go away from the fixed surface of contact. Now if $A = 1$ and $\frac{du}{dy} = 1$ then $|F| = \eta$ which is called coefficient of viscosity.

Let $F = -\eta A \frac{du}{dy}$ and $\eta = -\frac{F}{A \cdot \frac{du}{dy}}$

Hence the viscous force per unit area per unit ~~[illegible]~~ velocity gradient will be equal to $\frac{1}{3} m n \bar{c} \lambda$. By definition this is equal to the coefficient of viscosity η.

∴ The coefficient of viscosity of gas

$$\boxed{\eta = \frac{1}{3} m n \bar{c} \lambda} \longrightarrow ④ \Rightarrow \boxed{\eta = \frac{1}{3} \rho \bar{c} \lambda} \longrightarrow ⑤$$

where $\rho = mn$ = the density of the gas.

But we know that $\lambda = \frac{1}{\sqrt{2} \cdot \pi a^2 n}$

$$\therefore \boxed{\eta = \frac{m \bar{c}}{3\sqrt{2} \cdot \pi a^2}} \longrightarrow ⑥$$

The coefficient of viscosity η of a gas is

① Directly proportional to the mass m

② " " " average velocity of $\bar{c}$

Then $\bar{c} \propto \sqrt{T}$

⑤ Inversily proportional to square of diameter (a)

$\lambda = \frac{KT}{\sqrt{2} \pi P a^2}$. But coeff of viscosity is independent of the pressure and density of gas.

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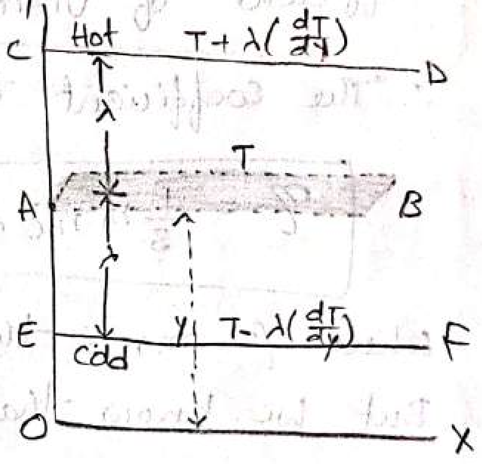
→ Thermal Conductivity :-

The transport of heat energy from one region of the gas that is at high temp to the region of lower temp gives rise to the transport phenomenon of thermal conduction.

It is explained on the basis of kinetic theory of gases like viscosity.

Let us consider that the gas to be at rest and divided into several layers. The molecule of one layer differ from those of the other layer in respect of temp. There exists a uniform temp gradient $\left(\frac{dT}{dy}\right)$ in the gas.

Let the layer OX be the coldest and the temp gradually increases ~~incases~~ in OY direction as shown in fig.



Consider a layer AB at a distance y from OX which is at a temp T. There are two layers CD and EF on either side of AB at a distance λ, whose ~~than~~ temps are $\left(T+\lambda\frac{dT}{dy}\right)$ and $\left(T-\lambda\frac{dT}{dy}\right)$ respectively.

∴ The phenomenon of thermal conduction is due to the transport of energy. The molecules they above AB plane is at high temp than below AB. The molecules from CD are passing downwards across AB will posses ~~ket~~ kinetic energy is more when compare to molecules comming from EF which are in upward direction.

Expression for thermal conductivity :-

Let us consider one-third of molecules are moving on each of the three directions parallel to x, y, and z axes. Now no of molecules crossing the layer AB in one direction (up & down) per unit ~~area~~ area per second can be written as

$$\left(\frac{n\bar{c}}{6}\right)$$

where n is the no of molecules per unit-volume of the gas and $\bar{c}$, the average velocity of molecules.

If m be the mass of the molecules the total mass crossing unit area of AB per second either side is given by $(mn\bar{c}/6)$

The total heat energy Q_1 carried by all the molecules in crossing layer AB downwards per unit area per second is given by

$$Q_1 = \text{mass} \times \text{specific heat} \times \text{temp.}$$

$$= \left(\frac{mn\bar{c}}{6}\right) C_v \left[T + \lambda\frac{dT}{dy}\right] \longrightarrow ①$$

Illy total heat energy Q_2 carried by all the molecules in crossing the layer AB upwards per unit area per second is given by

$$Q_2 = \left(\frac{mn\bar{c}}{6}\right) C_v \left[T - \lambda\frac{dT}{dy}\right] \longrightarrow ②$$

∴ Now transfer of energy $Q = Q_1 - Q_2$.

&

$$Q = \left(\frac{mn\bar{c}}{6}\right) C_v \left[T + \lambda\frac{dT}{dy}\right] - \left(\frac{mn\bar{c}}{6}\right) C_v \left[T - \lambda\frac{dT}{dy}\right]$$

$$= \left(\frac{mn\bar{c}}{6}\right) C_v \left[\left(T + \lambda\frac{dT}{dy}\right) - \left(T - \lambda\frac{dT}{dy}\right)\right]$$

$$= \frac{1}{3} mn\bar{c}\, C_v\, \lambda\frac{dT}{dy} \longrightarrow ③$$

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The coefficient of thermal conductivity of a gas is defined as the quantity of heat that flows per unit area per second per unit temp gradient and is given by

$$Q = K\left(\frac{dT}{dy}\right) \quad \& \quad K = \frac{Q}{(dT/dy)} \longrightarrow ④$$

Substitute the value of Q, we get

$$K = \frac{\frac{1}{3} mn\bar{c}\, C_v \lambda \left(\frac{dT}{dy}\right)}{\left(\frac{dT}{dy}\right)} = \frac{1}{3} mn\bar{c}\, C_v \lambda$$

$$\therefore \boxed{K = \frac{1}{3} \rho \bar{c}\, C_v \lambda} \longrightarrow ⑤$$

Where (ρ is the density of gas)

we know that $\lambda = \frac{1}{(\sqrt{2})\pi d^2 n}$

$$\therefore K = \frac{1}{3} mn\bar{c}\, C_v \left(\frac{1}{\sqrt{2}\,\pi d^2 n}\right)$$

$$\boxed{K = \frac{1}{3\sqrt{2}} \frac{m\bar{c}\, C_v}{\pi d^2}} \longrightarrow ⑥$$

which is the required expression for coefficient of thermal conductivity of a gas.

→ **Diffusion of gases:-** when there is a single gas in vessel the process of diffusion takes place.

It is mass transfer inside a gas from high concentration to lower concentration region. The Diffusion Coefficient D of a gas defined as no of molecules of the gas crossing per unit surface area per second per unit concentration gradient. The no of molecules J_y diffusing through " " $\frac{dn}{dy}$ in a time t through the surface A directly proportional to A (area) and concentration gradient $\frac{dn}{dy}$

$$J_y = -D.A\frac{dn}{dy} \longrightarrow ⑦$$

D diffusion const.

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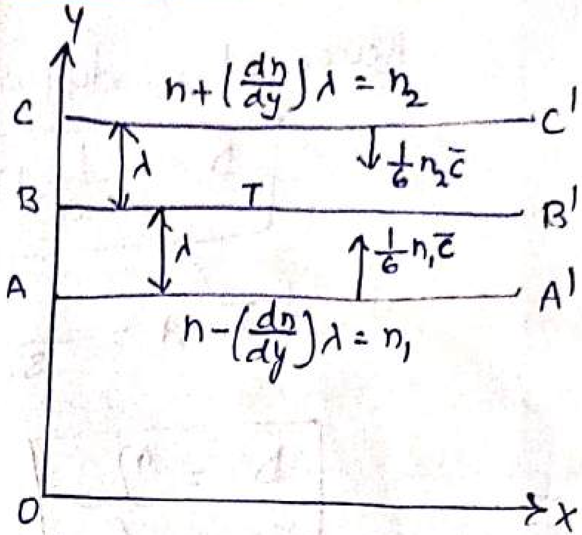
Let us consider a region BB' of the gas that is parallel to ox axis as shown in fig. Let the concentration of the gas in this region be η. The concentration in this cc' region will be $n+\left(\frac{dn}{dy}\right)\lambda$.



Illy in the region AA' is at distance λ below BB' will be $n-\left(\frac{dn}{dy}\right)\lambda$. Then the no of molecules diffusing in unit time for unit surface area from cc' layer to BB' layer is

$$J_1 = \frac{1}{6}\left[n+\left(\frac{dn}{dy}\right)\lambda\right]\bar{c} \longrightarrow ②$$

④

from AA' layer to BB' layer is

$$J_2 = \frac{1}{6}\left[n-\left(\frac{dn}{dy}\right)\lambda\right]\bar{c} \longrightarrow ③$$

The diffusion takes place in y-direction and hence J_2 is +ve and J_1 is -ve. Then the no of molecules diffusing per unit surface area in one second will be

$J_y = J_2 - J_1$ that is

$$J_y = \frac{1}{6}\left[-2\left(\frac{dn}{dy}\right)\lambda\right]\bar{c} = -\frac{1}{3}\left(\frac{dn}{dy}\right)\lambda\bar{c}$$

The no of molecules diffusing through a surface area A in time t will be

$$J_y = -\frac{1}{3}A\left(\frac{dn}{dy}\right)\lambda\bar{c}t \longrightarrow ④$$

$$= -DA\left(\frac{dn}{dy}\right)t \quad \text{from eq ①}$$

Hence the diffusion coefficient

$$\boxed{D = \frac{1}{3}\lambda\bar{c}} \longrightarrow \text{⑤}$$

But $\frac{1}{3}\bar{c}\lambda = \frac{1}{3}\frac{\bar{c}\lambda\rho}{\rho} = \frac{\eta}{\rho}$ and hence

$$\boxed{D = \frac{\eta}{\rho}} \longrightarrow \text{⑥}$$

Again $\lambda \propto \frac{1}{n} \propto \frac{T}{P}$ and $\bar{c} \propto \sqrt{T}$ and hence

$$\boxed{D \propto \frac{T^{3/2}}{P}} \longrightarrow \text{⑦}$$

Thus, the diffusion coefficient D is

1) Directly proportional to $T^{3/2}$ and
2) Inversely proportional to the pressure P.

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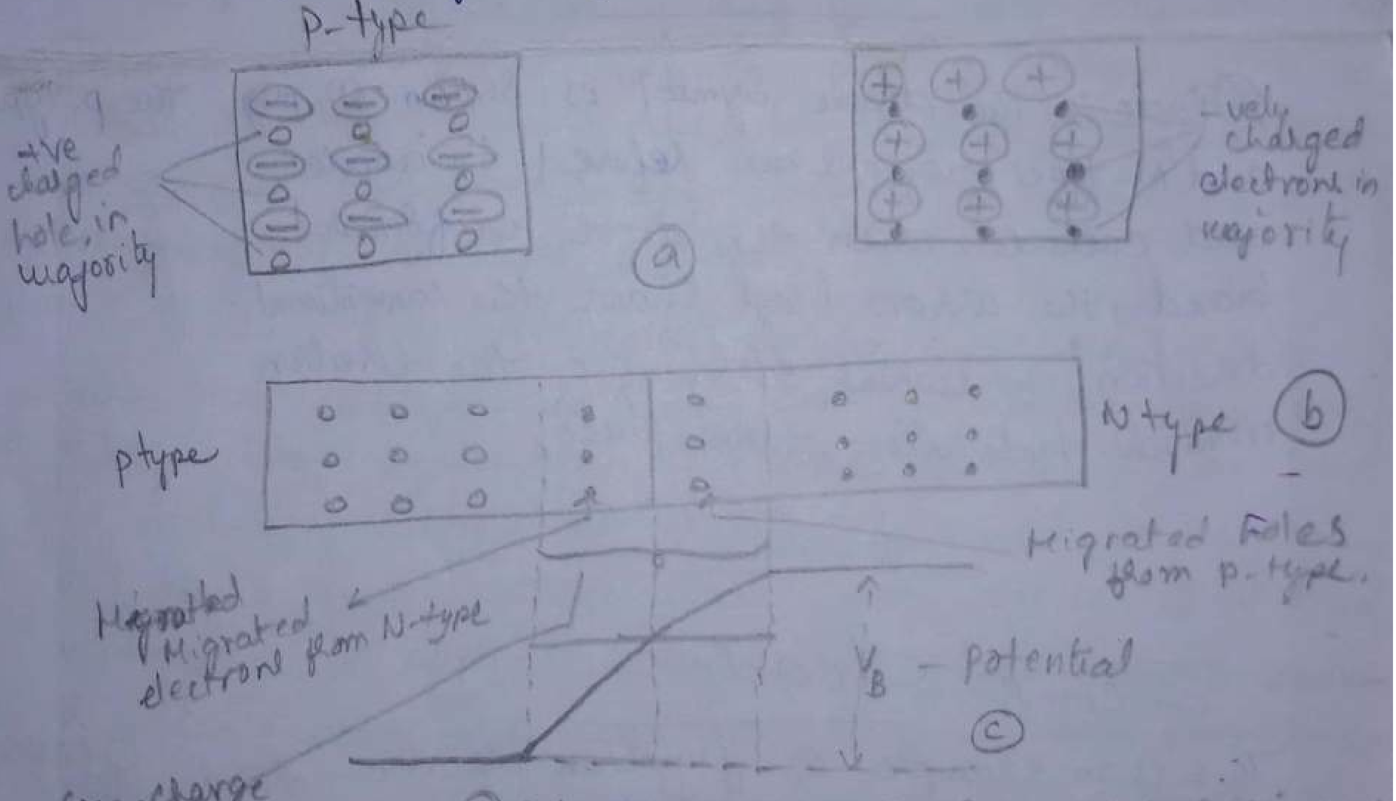
7. Basic Electronics.

①

→ P-N junction diode:-

This is a two terminal device consisting of a P-N junction formed either in Ge & Si crystal, when a P-type material is intimately joined to N-type a P-N junction is formed. A P-N junction is formed from a piece of semiconductor (Ge) by diffusing P-type material to one half side and N-type material to other half side. The plane dividing the two zones is known as junction.

A P-N junction is illustrated in fig ①. fig ①ⓐ shows P-type and N-type semiconductor pieces before they are joined.

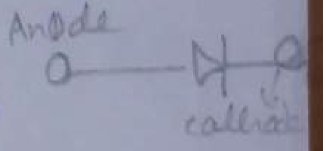


ⓐ P-type, N-type semiconductor (not joined)
ⓑ P-N junction (after joining)
ⓒ potential barrier.

As p-type material has a high concentration of holes and N-type material has high concentration of free electrons. Hence there is a tendency of holes to diffuse over to N-side and electrons to P-side, The process is known as diffusion.

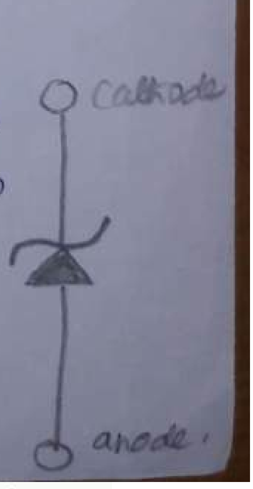
② Due to diffusion some holes from P side cross over to N-side where they combine with electrons become neutral. and vice versa. Thus a region is formed which is known as depletion layer & charged free region & space charge region. The diffusion of holes and electrons continues till a potential barrier fig ①ⓒ is developed in space charge region & charge free region. which prevent further diffusion & neutralization. The potential barrier can be increased & decreased by applying an extranal voltage.

Diode:- The diode symbol is shown in fig. The p-type and N-type regions are refered to anode and cathode. when the diode is forward biased, the arrow head shows the conventional direction of current flow. i-e the direction in which hole flow takes place.



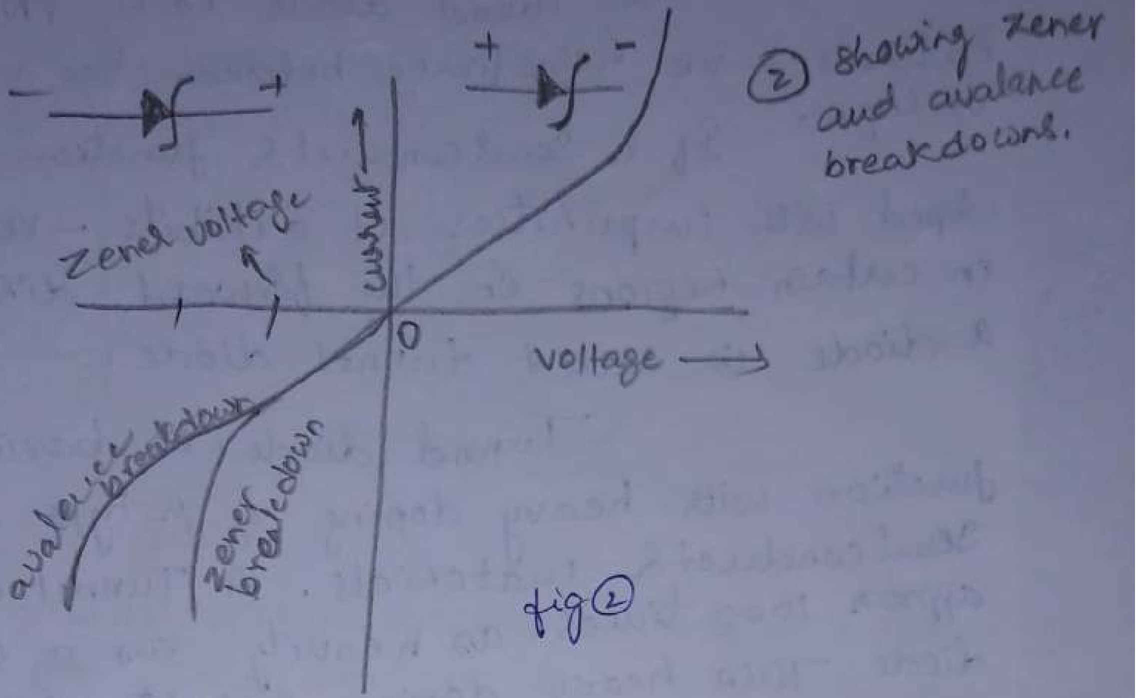
→ Zener diode:-

This is an example of junction device. Zener diode is a reverse biased heavely doped silicon (ge) P-N junction diode, which is operated in the breakdown region. The symbol of Zener diode is shown in fig①. This is similar to normal diode expect the line representing the cathode



is bent at both ends. i-e like the letter Z for ② Zener. The V-I characteristics is shown in fig②.

③



② showing zener and avalance breakdowns.

fig②

From fig ② the reverse voltage applied to P-N junction is increased from zero, the current remains very small over a long range and increases very slightly with increasing bias. But when the reverse bias is made very high the covalent bonds near the junction breakdown. This corresponding voltage is called break down voltage & Zener voltage.

In this region of characteristic curve the voltage across the diode remains const over a large range of current. The Zener diode may be used to stabilize voltage at a predetermined value. The diff b/w a junction diode and Zener diode is external voltage can be applied either under forward bais condition & under reverse bias condition while to Zener diode reverse bias is only applied and operated in break-down region.

→ Tunnel diode :-

A Tunnel diode is a PN junction that exhibits -ve resistance between two values of forward voltage. If a semiconductor junction diode is heavily doped with impurities, it exhibits -ve resistance in certain regions in the forward direction. Such a diode is called tunnel diode.

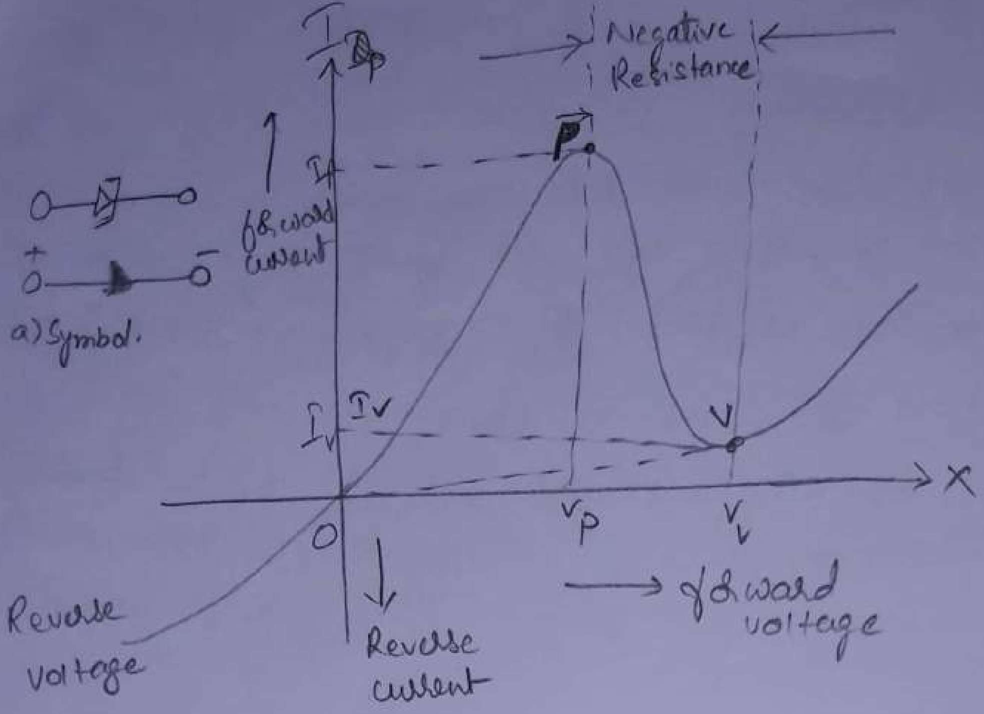
Tunnel diode is basically a PN junction with heavy doping of p-type and n-type semiconductor materials. A Tunnel diode is doped approx 1000 times as heavily as a conventional diode. This heavy doping results in a large no of majority carriers. As a result depletion layer is very narrow (i.e 100 times narrower). The operation of a tunnel diode depends upon the tunneling effect.

Tunneling effect:- The movement of valence electrons from the valence energy band to the conduction band with little & no applied forward voltage is called ~~tunne~~ tunneling. valence electrons seem to tunnel through the forbidden energy band.

As the forward voltage is first increased the diode current rises rapidly due to tunneling effect. Soon the tuneling effect is reduced the current flow starts to decrease as the forward voltage across the diode is increased. The tunnel diode is said to ~~the~~ have entered the -ve resistance region.

V-I characteristics :- fig ⓐ shows the V-I characteristics of a typical tunnel diode.

⑤



1) As the forward voltage across the tunnel diode is increased from zero, electrons from the n-region tunnel through the potential barrier to the P-region.

As the forward voltage increases until the peak point P reached. The diode current has now reached peak current I_p at about peak point voltage V_p. Now diode has exhibited +ve resistance.

2) As the voltage is increased beyond V_p, the tunneling action starts decreasing and the diode current decreases as the forward voltage increases until valley point V is reached.

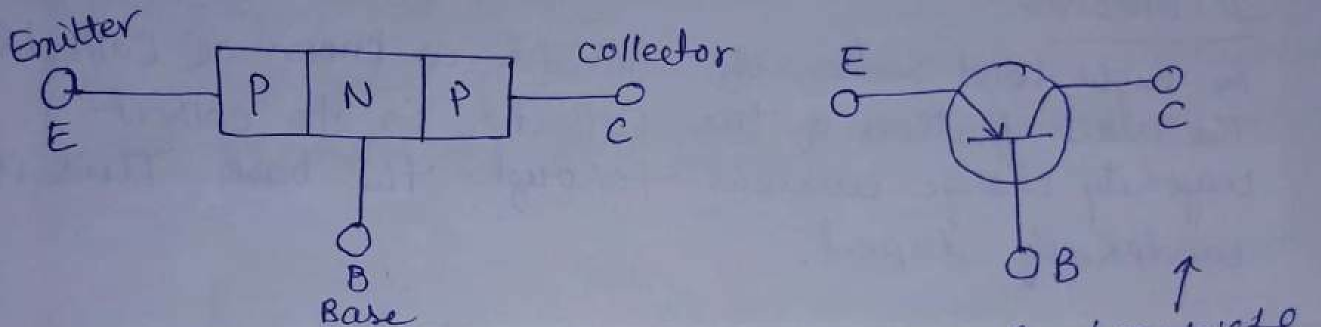
At this point tunnel diode when operated in the -ve resistance region, can be used as an oscillator & a switch.

3) When forward bias is increased beyond valley point, voltage V_v the tunnel diode behaves as a normal diode. the

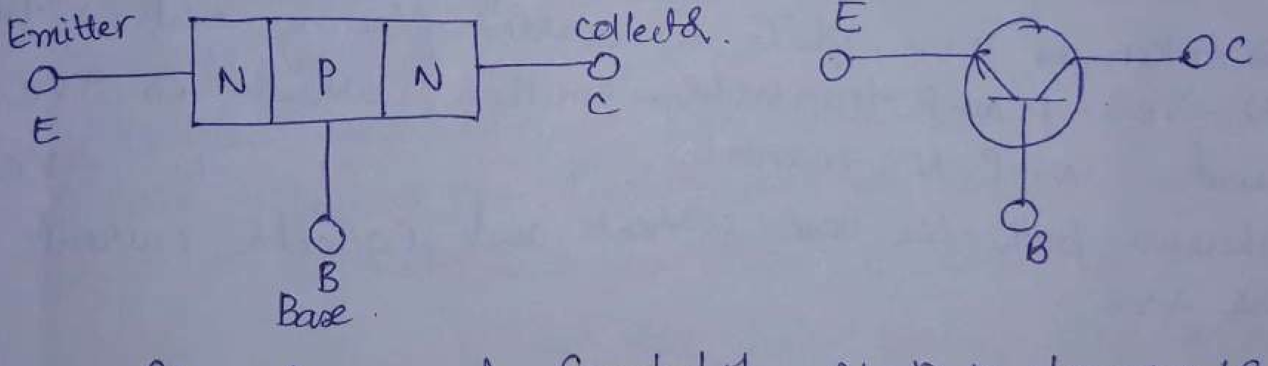
→ PNP and NPN transistors:-

A junction transistor is a sandwich of one type of semiconductor material b/w two layers of the other type. There are two types of transistors.
① P-N-P transistor ② N-P-N transistor.

⑥ When a layer of N-type material is sandwiched b/w two layers of P-type material, the transistor is known as P-N-P transistor. This is shown in fig ⓐ.



Structure and symbol of P-N-P transistor.



Structure and Symbol of N-P-N transistor.

When a layer of P-type material is sandwiched between two layers of N-type material the transistor is known as N-P-N transistor.

The transistor is made either from silicon & germanium crystal. The symbols are also shown in fig. A transistor (P-N-P & N-P-N) has the following

Sections.

① **Emitter**:-

This is left hand section of the transistor. The main function of this region is to supply majority charge carriers to the base and hence it is more heavily doped in comparision of other regions

② **Base**:-

The middle section of the transistor is known as base. This is very lightly doped and is very thin (10^{-6} m) as compared to either emitter or collector. So that it may pass most of the injected charge carriers to the collector

⑨

③ **Collector**:-

The right hand section of transistor is known as collector. The main function of the collector is to collect majority charge carriers through the base. This is moderatly doped.

→ **Sign Conventions**:-

A current entering into the transistor is taken as +ve, while a current flowing out is taken as −ve. P-N-P transistor – emitter current is +ve and N-P-N transister " " " −ve. whereas both the base current and collector current are +ve.

Fig

→ Operation of P-N-P transistor

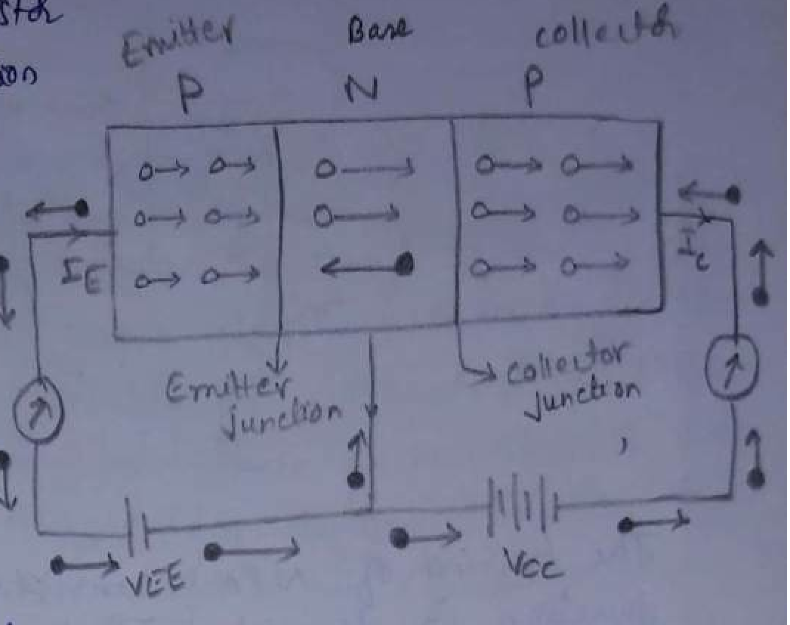


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fig shows a PNP transistor with Emitter-base junction as forward biased and Collector base junction as reverse biased. The PNP transistor operation is as follows. The holes of p region are repelled by +ve terminal of battery V_{EE} towards the base. The potential barrier is reduced and is forward biased the holes are penetrate into N-region, This constitute emitter current I_E.

The holes are able to drift across the base and swept by -ve Collector voltage V_{CC} and constitute collector current I_C.

Now the co-valent bond near the emitter breaks down, The holes immediatly move towards the emitter junction.

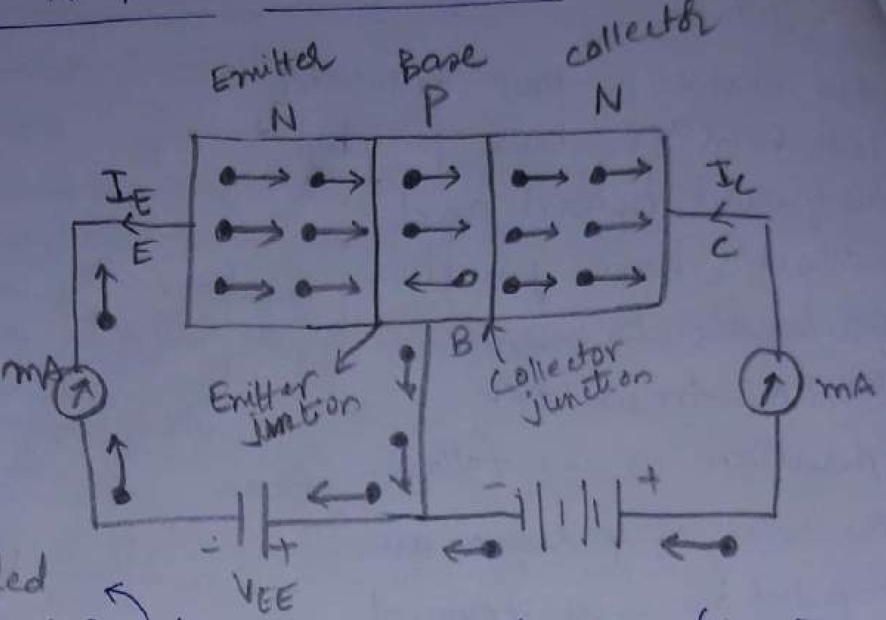
① current conduction in PNP transistor is by holes
② Collector current is slightly less than emitter current.
③ collector current is a function of emitter current.

Thus the hole current plays an important role in operation of PNP transistor.

$$\therefore \boxed{I_E = I_B + I_C}$$

→ Operation of N-P-N transistor:-

⑨



electrons repplled

The baising of NPN transistor is shown in fig. The emitter junction is forward biased and collector junction is reverse biased because electrons are flowing away from collector junction.

The electrons repelled from -ve terminal of battery towards emitter junction, the potential barrier reduced due to forward baised Now the electrons in N-region (collector) swept up by Vcc and electron conduction takes place continuously so long as the two junctions are properly biased. So the current conduction in NPN transistor is carried out by electrons.

$$\boxed{I_E = I_B + I_C}$$

→ Transistor circuit configurations :-

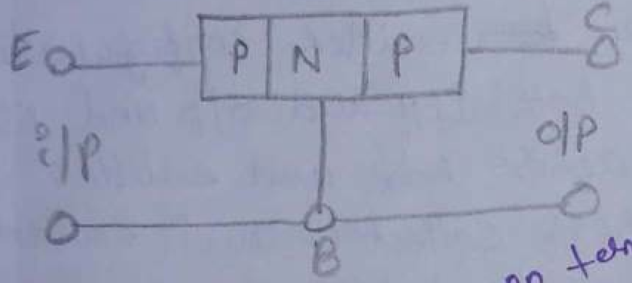
Following are the three types of transistor ckt configuration
① common- base (CB)
② common- emitter (CE)
③ common- collector (CC)

10

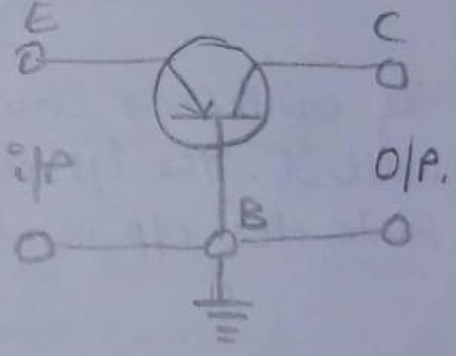
Here common is used to denote the transistor lead which is common to the i/p and o/p ckt. This is because when transistor is connected in a ckt four terminals are required, while a transistor has only three terminals. This difficulty is removed by making one terminal of the transistor common to both i/p and o/p terminals.

The common terminal is generally grounded. Each ~~fo~~ configuration has specific advantage and disadvantage.

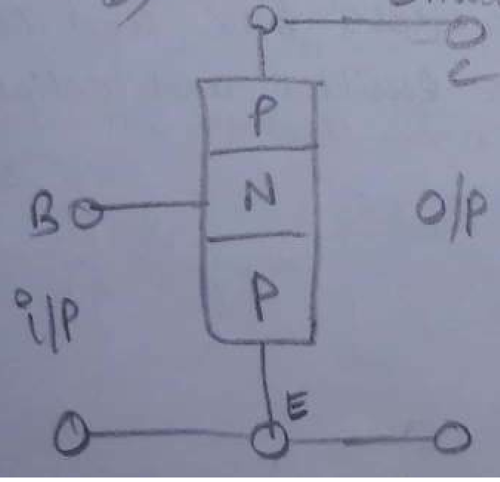
a) common base (CB) configuration

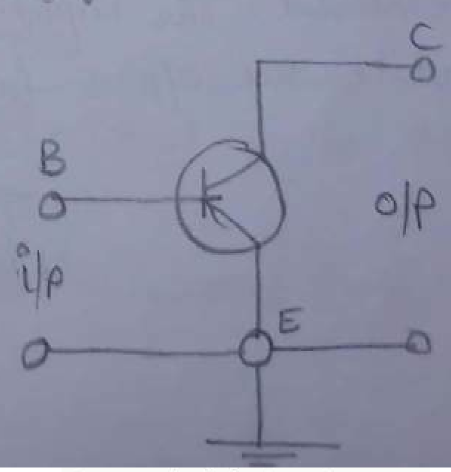


common terminal is grounded.

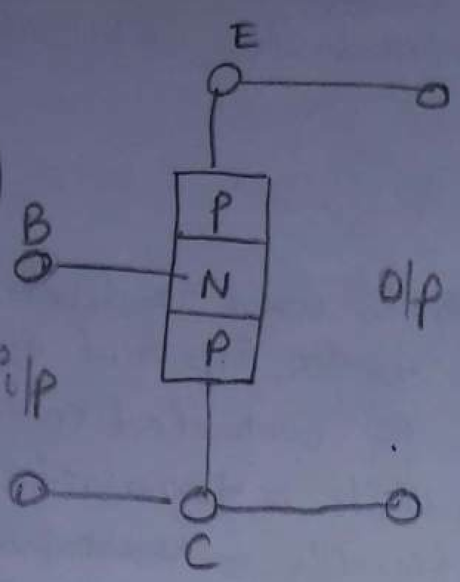


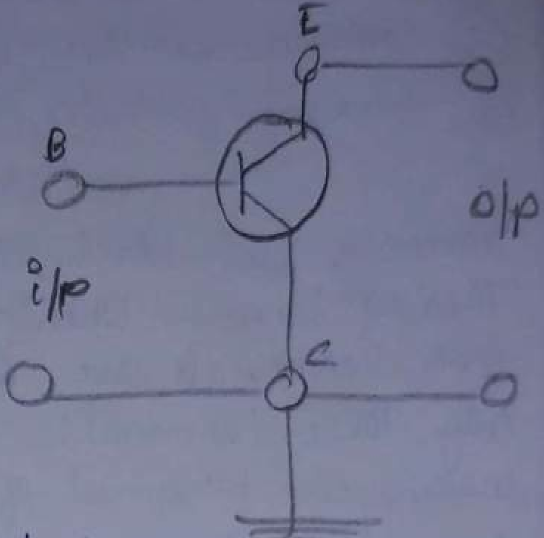
b) Common Emitter (CE) Configurations





c) Common Collector (CC) Configurations





The emitter is always forward baised while the collector is always reverse biased. The different configurations of a PNP transistor are shown in fig.

In common base configuration the base is common to both i/p and o/p and is earthed. The i/p is given across emitter and base while the o/p is ~~ta~~ taken across collector ~~&~~ and base.

In Common ~~bas~~ emitter configuration the emitter is common to both i/p and o/p and is earthed. The i/p is given across base and emitter while the o/p is taken across collector and emitter.

In common collector configuration, the collector is common to both i/p and o/p and is earthed. The input is given across base and collector while the o/p is taken across emitter and collector.

→ Assignment diff b/w CB, CE, CC Configurations

→ Hybrid parameters & h-parameters :-

Hybrid means mixed. The parameters have mixed dimensions and hence they are called as hybrid parameters. These parameters are very useful at high frequencies. This is due to that fact that at high frequencies a transistor has low input impedence and high output impedance

⑫ At high frequencies the Y and Z parameters becomes awkward. In this case i_1 and v_2 taken as independent variables. while v_1 and i_2 are taken as dependent variables.

~~$V_1 = f(i_1, v_2)$ → ① independent~~

~~$i_2 = f(i_1, v_2)$ → ② dependent.~~

~~$V_1 = f(i_1, v_2)$~~

~~V~~

$$v_1 = f(i, v_2) \longrightarrow ① \text{ independant}$$

$$i_2 = f(i, v_2) \longrightarrow ② \text{ dependent}$$

Taking the total differentials we get

$$dv_1 = \frac{\partial v_1}{\partial i_1} di_1 + \frac{\partial v_1}{\partial v_2} dv_2 \longrightarrow ③$$

$$di_2 = \frac{\partial i_2}{\partial i_1} di_1 + \frac{\partial i_2}{\partial v_2} dv_2 \longrightarrow ④$$

Again for small a-c signals limited to the quasi linear region the partial derivaties becomes constants. Thus

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow ⑤$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow ⑥$$

In the matrix form the eq's ⑤ and ⑥ can be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \longrightarrow ⑦$$

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By putting $I_1 = 0$ (open ckt) and Short ckt ($V_2 = 0$) in eq ⑤ and ⑥ we can define the small signal a-c values of h-parameters, If we short cut the output terminals $V_2 = 0$ then short ckt.

⑬

$V_1 = h_{11} I_1 + h_{12} 0$ ∴ $h_{11} = \frac{V_1}{I_1}$

$I_2 = h_{21} I_1 + h_{22} 0$ $h_{21} = \frac{I_2}{I_1}$

If we open ckt. the input terminals i.e $I_1 = 0$ then

$V_1 = h_{11} 0 + h_{12} V_2$ ∴ $h_{12} = \frac{V_1}{V_2}$

$I_2 = h_{21} 0 + h_{22} V_2$ $h_{22} = \frac{I_2}{V_2}$.

So ~~h-parameters are defined as~~

$h_{11} = h_i = \left.\frac{\partial V_1}{\partial I_1}\right|_{V_2 = 0} = \frac{V_1}{I_1}$ = i/p impedence with o/p short circuited $V_2 = 0$

$h_{12} = h_r = \left.\frac{\partial V_1}{\partial V_2}\right|_{I_1 = 0} = \frac{V_1}{V_2}$ = Reverse voltage ratio with i/p open circuited $I_1 = 0$.

$h_{21} = h_f = \left.\frac{\partial I_2}{\partial I_1}\right|_{V_2 = 0} = \frac{I_2}{I_1}$ = forward current gain with o/p short circuited $V_2 = 0$

$h_{22} = h_o = \left.\frac{\partial I_1}{\partial V_2}\right|_{I_1 = 0} = \frac{I_2}{V_2}$ = o/p admittance with i/p open ckted $I_1 = 0$.

Hence $\boxed{h_i, h_r, h_f, h_o}$ are the hybrid parameters.

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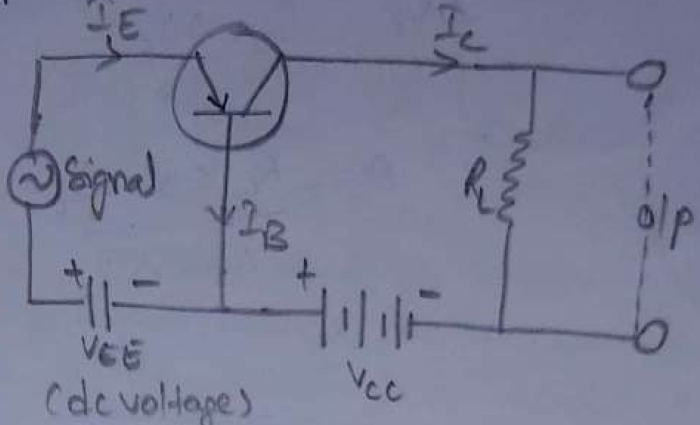
→ Transistor As an Amplifier :-

fig shows the basic ckt of a transistor amplifier. The weak signal to be amplified is applied b/w emitter-base ckt and o/p is taken across the Load Resistor R_L. connected in the collector ckt.



A d.c voltage V_{EE} is also connected in the i/p ckt (~~In emitter bias battery V_{EE} of such a magnitude that i/p ckt is always forward biased regardless of the polarity of the signal is connected~~) when collector current flows through the load resistance R_L a large voltage is developed across it.

Let a small voltage change ΔV_i b/w emitter and base causes a relatively large emitter current change ΔI_E. Let α be the fraction of this current change which is collected and passed through R_L.

$$\boxed{\alpha = \frac{\Delta I_C}{\Delta I_E}}$$

$$\Delta I_C = \alpha \cdot \Delta I_E$$

The change in o/p voltage across the load resistor

$$\Delta V_0 = R_L \times \Delta I_C$$

$$= R_L \times \alpha \Delta I_E$$

Under these circumstances the voltage amplification

$$A = \frac{\Delta V_0}{\Delta V_i} > 1$$

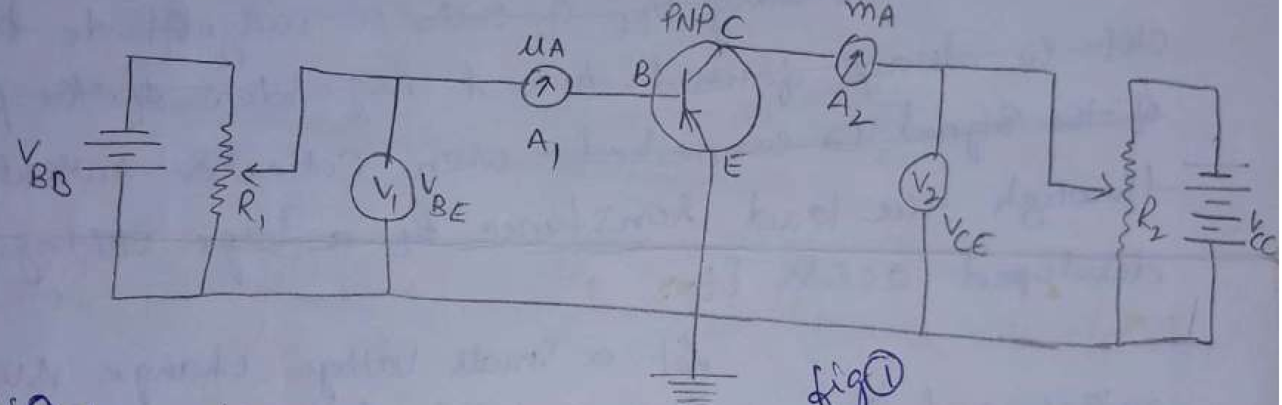
will be greater than unity and the transistor acts as an amplifier. If the dynamic resistance of the emitter junction be r_e then

$$\Delta V_i = r_e \cdot \Delta I_E$$

(15)

$$\therefore A = \frac{R_L \times \alpha \cdot \Delta I_E}{r_e \cdot \Delta I_E} = \frac{\alpha \cdot R_L}{r_e}$$

→ Transistor (CE) characteristics



fig①

fig① shows the ckt arrangement for ploting the static characteristic of a PNP transistor in common emitter configuration.

In ckt the battery V_{BB} provides forward bias to ~~mi~~ emitter base junction with the help of potential devider R_1. The voltmeter V_1 measures the base-emitter voltage V_{BE}. The micro Ammeter A_1 measures the base current I_B. A battery V_{CC} is connected b/w collector and emitter through a potential divider R_2.

The +ve terminal of battery is connected to the ~~collector~~ emitter while -ve " " " collector so that the collector is Reverse biased. The voltmeter V_2 measures the collector-emitter voltage V_{CE} and the milli-ammeter A_2 measures the collector current.

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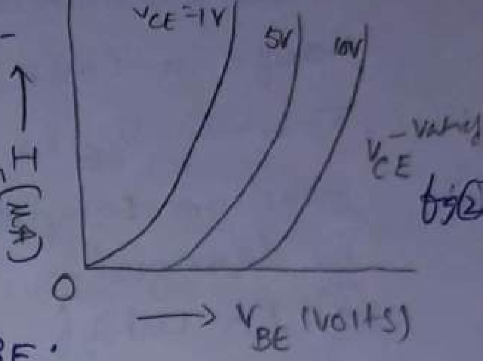
i/p characteristics :- The curve b/w base current I_B and base-emitter voltage V_{BE} at const collector-emitter voltage V_{CE} represents the i/p characteristics. For ploting V_{CE} is kept fixed V_{BE} is varied with help of R_1, the base current I_B is noted for each value of V_{BE}. A graph of I_B aganist V_{BE} is drawn, and is shown in fig ②



fig②

⑯ From this ① It is a forward-biased diode curved and the base emitter section of transistor is a diode and it forward biased.

② I_B increases less rapidly with V_{BE} as compared to common base configuration.

This shows that the input resistance of common-emitter circuit is higher than that of common-base circuit.

Output characteristics :- The curve b/w collector current I_C and collector emitter voltage V_{CE} at constant base current I_B represents the output characteristic. For ploting I_B is kept fixed with the help of R_1, the value of V_{CE} is varied in steps and collector current I_C is noted for each value of V_{CE}.

A graph of I_c against V_{CE} is drawn. The curve so obtained is known as o/p characteristics. For diff values the graph is shown in fig ③ from graph

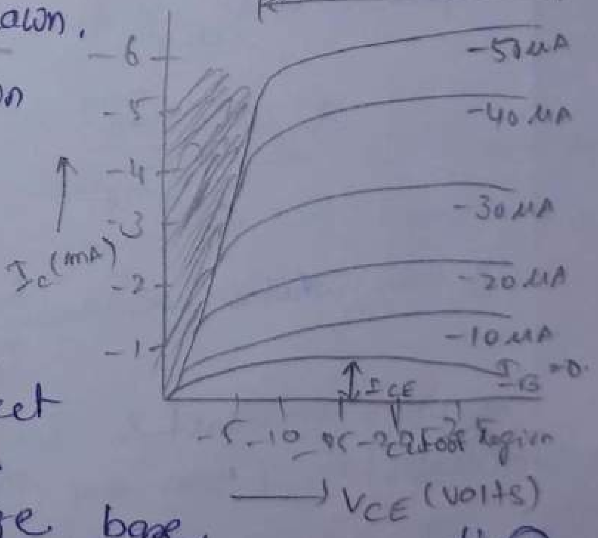


fig ③

① In the active region for small values of base current the effect of collector voltage over collector current is small while for large base current values this effect increases.

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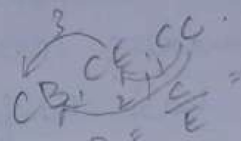
② when V_{CE} is very low value, the transistor is said to be Saturated., In this The base current I_B does not produce change in Collector current I_C.

③ when V_{CE} is allowed to increase too far collector-base junction completly breaksdone. due to this the transistor is damaged.

④ In cutoff region a small amount of collector current flows even when base current $I_B = 0$. This is called I_{CEO}. Then the transistor is said to be cutoff.

⑰

→ Relation b/w α, β, γ :-



$CB = \frac{C}{E} = \alpha$

$CE = \frac{C}{B} = \beta$

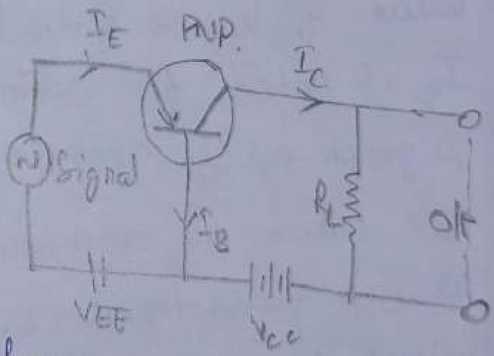
$CC = \frac{E}{B} = \gamma$

① Common-base (CB) Configuration :- ~~(CE)~~

In this configuration i/p signal is applied b/w emitter and base while the o/p is taken from collector and base. As base is common to i/p and o/p ckt's, hence the name common base configuration. fig ⑧ shows the common base PNP transistor ckt.

Current amplification factor (α) :-

When no signal is applied then the ratio of the collector current to the emitter current is called dc alpha (α_{dc}) of a transistor.



$$\alpha_{dc} = \frac{-I_C}{I_E}.$$

{ I_E flows into transistor (-ve) }
{ I_C flows out of it }

if we write $\alpha = \frac{-I_C}{I_E}$ ⟶ ①

α of a transistor is a measure of the quality of a transistor. from eq ①

we know that $\boxed{I_E = I_B + I_C}$

$I_C = \alpha I_E$ and hence $I_B = I_E - I_C$.

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∴ ~~$I_B = I_C - \alpha I_E = I_E(1-\alpha)$~~.

$$\therefore I_B = I_E - \alpha I_E = I_E(1-\alpha) \longrightarrow ②$$

When signal is applied the ratio of change in collector current to the change in emitter current at constant collector base voltage is defined as current amplification factor

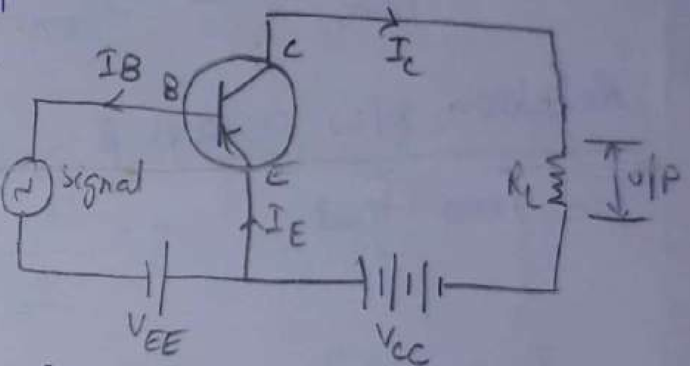
⑱

$$\alpha_{ac} = -\frac{\Delta I_C}{\Delta I_E} \longrightarrow ③$$

For physical purpose $\alpha_{dc} - \alpha_{ac} - \alpha$ and practical values in commertial transistor range from 0.9 to 0.99.

② Common-emitter (CE) configuration :-

In this configuration the i/p signal is applied b/w base and emitter and the o/p is taken from collector and emitter, emitter is common to i/p & o/p ckts and shown in fig ② which shows the common-emitter PNP transistor CKt.



Base current amplification factor :- (β)

~~$\beta_{dc} = \beta = \frac{I_C}{I_B} \longrightarrow ④$~~

When no signal is applied then the ratio of collector current to be base current called dc beta (β_{dc}) of transistor

$$\beta_{dc} = \beta = \frac{I_C}{I_B} \longrightarrow ④$$

When signal is applied $\beta_{ac} = \beta = \frac{\Delta I_C}{\Delta I_B} \longrightarrow ⑤$

from eq ④ $I_C = \beta I_B \longrightarrow ⑥$

$(1-\alpha)$

Relation b/w α and β :-

we know that $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$

$I_E = I_B + I_c$ & $I_B = I_E - I_c$

NOW $\beta = \frac{I_C}{I_E - I_c} = \frac{I_c/I_E}{1-(I_c/I_E)} = \frac{\alpha}{1-\alpha}$. ⑧

from eq ⑧ we get

$\beta(1-\alpha) = \alpha$ &

$\beta - \beta\alpha = \alpha$

$\beta = \alpha(1+\beta)$

$\therefore \alpha = \frac{\beta}{1+\beta}$ ⑨

$1-\alpha = 1 - \frac{\beta}{1+\beta}$

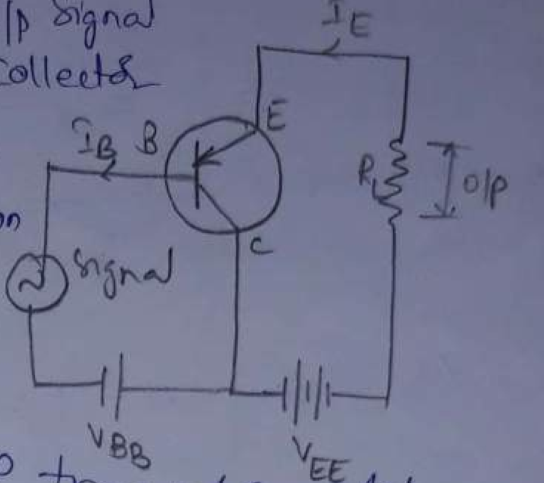
$= \frac{1+\cancel{\beta}-\cancel{\beta}}{1+\beta} = \frac{1}{1+}$

$\boxed{1-\alpha = \frac{1}{1+\beta}.}$

It can be sean that $1-\alpha = \frac{1}{1+\beta}$ ⑩

③ Common - Collector (CC) Configuration:-

In this configuration, the i/p signal is applied b/w base and collector and o/p is taken from the emitter. As collector is common to i/p and o/p ckt, the name common collector configuration is shown PNP transistor ckt.



current amplification factor (γ):-

When no signal is applied then the ratio of emitter current to the base current is called as dc gamma (γ_{dc}) of the transistor.

⑳

$$\gamma_{dc} = \gamma = \frac{I_E}{I_B} \longrightarrow (11)$$

When signal ~~signal~~ is applied $\gamma = \dfrac{\Delta I_E}{\Delta I_B} \longrightarrow (12)$

We know that $I_E = I_B + I_C$

Also $I_C = \alpha I_E + I_{CEO}$

$$I_E = I_B + (\alpha I_E + I_{CBO})$$

$$= I_B + \alpha I_E + I_{CBO}$$

$$I_E(1-\alpha) = I_B + I_{CBO}.$$

$$I_E \simeq \frac{I_B}{(1-\alpha)} + \frac{I_{CBO}}{(1-\alpha)}$$

$$I_E = (1+\beta) I_B + (1+\beta) I_{CEO} \longrightarrow (13)$$

from $\therefore \dfrac{1}{1-\alpha} = (1+\beta)$

we know that $\gamma = \frac{I_E}{I_B}$ and $\alpha = \frac{I_C}{I_E}$.

$$I_B = I_E - I_C$$

Now $\gamma = \frac{I_E}{I_E - I_C} = \frac{1}{1-(I_C/I_E)} = \frac{1}{(1-\alpha)} \rightarrow ⑭$

we know that $(1-\alpha) = \frac{1}{1+\beta}$

Substitute in eq ⑭ ~~in eq~~

㉑

$$\boxed{\gamma = \frac{1}{(1-\alpha)} = (1+\beta)}$$

CB configuration	CE configuration.	CC configuration
① The i/p current is the emitter current I_E and the o/p current is the collector current I_C.	① The i/p current is the base current I_B and o/p current is collector current I_C	① The i/p current is the base current I_B and the o/p current is the emitter current I_E.
② $\alpha = \frac{\Delta I_C}{\Delta I_E}$ at V_{CB} const current amplification factor.	② The current amplification factor $\beta = \frac{\Delta I_C}{\Delta I_B}\Big\vert_{at\ V_{CE}=const}$	② current amplification factor $\gamma = \frac{\Delta I_E}{\Delta I_B}$
③ This configuration provides voltage gain but no current gain	③ This configuration provides good current gain and voltage gain.	③ This configure provides good current gain and voltage gain is always less than 1.
④ V_{CB} - const, small increase in V_{EB}, I_E gets increased.	④ V_{CE} - const with small increase in V_{BE}, the base current I_B increases rapidly than in CB configuration	④ r_i is high r_o is low.
⑤ I_E is independent of collector voltage V_{CB}	⑤ for any value of V_{CE}, I_C is equal to βI_B.	⑤ sum of collector current and base current equals emitter current.
⑥ When V_{EB} is kept const, V_{CB} can effect I_C only at low voltage	⑦ $r_i = \frac{\Delta VBE}{\Delta I_B}\Big\vert_{at\ V_{CE}=const}$	⑥ The i/p and o/p signals are in phase.
⑦ $r_i = \frac{\Delta V_{EB}}{\Delta I_E}\Big\vert_{V_{CB}=const}$ input resistance.	⑧ $r_o = \frac{\Delta V_{CE}}{\Delta I_C}\Big\vert_{at\ I_B=const}$	⑦ This configuration works as non inverting amplifier output.
⑧ $r_o = \frac{\Delta V_{CB}}{\Delta I_C}\Big\vert_{I_E=const}$ Output resistance	⑨ As the r_o of CE ckt is less than that of CB ckt.	⑧ This ckt is mostly used for impedence matching. That means to drive a low impedence load from a high impedance Source.
⑨ This configur ~~produces~~ provides good stability against increase in tem.	⑩ This configuration is usually used for bias stabilization methods and audio frequency applications.	
⑩ CB configur is used for high freq application	⑥ As i/p resistance is very low value a small value of V_{BE} is	

22

Paper V, Sem V

8. Digital electronics

UNIT – V.

→ Introduction :-

The digital electronics is a branch of electronics which deals the generation, processing and Storage of digital Signals. This helps in the analysis, design and construction of digital Systems. These Signals are called as digital Signals. These Signals are coded as 0 & 1 numbers which correspond to OFF & ON conditions.

The digital electronics was first invented by George Boolie. The algebra which is used in digital electronics is called as Boolean algebra. The 0's and 1's represented by a bit. In this we use different Types of number Systems.

Here the following four Systems of arithmetic are often used in digital electronics.

1) Decimal number System,
2) Binary number System.
3) octal number System.
4) Hexadecimal number System.

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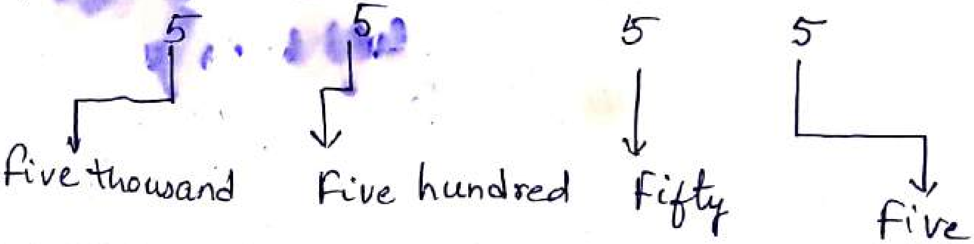
each digit position of a number.

→ The Delimal Number System:-

The delimal no System was developed by Aryan's around 5th century and now-a-days ~~to~~ used in our daily life. It contains 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. It has a base & ~~da~~ radix 10. Any digit may be used in each position of the no. The absolute values of each digit is fixed but its position value ~~of~~ & place value & weight is determined by its position in the overall no as shown below.



The characteristics of any number System are:-

1) base & radix is equal to the no of digits in the System.
2) the largest value of digit is one less than the radix and
3) each digit is multiplied by the base raised to appropriate power depending upon the digit position.

Eg:- $8765_{(10)} = 8\times\underset{\underset{1000}{\downarrow}}{10^3} + 7\times\underset{\underset{100}{\downarrow}}{10^2} + 6\times\underset{\underset{10}{\downarrow}}{10^1} + 5\times\underset{\underset{1}{\downarrow}}{10^0}$

5 - least significant digit (LSD)
8 - most ,, ,, (MSD)

Eg:- $\underset{2}{4}\underset{1}{3}\underset{0}{8}.\underset{-1}{5}\underset{-2}{2}\underset{-3}{8} = 4\times10^2 + 3\times10^1 + 8\times10^0 + 5\times10^{-1} + 2\times10^{-2} + 8\times10^{-3}$

Binary Number System:-

When we have to write any no. greater than 9, we combine 2nd ~~digit~~ no. in decimal and first ★ no. in decimal we get 10, like that 11, 12, 13, 14.... after 20, 21, 22, 23...... Similar logic follows for 100 which comes after 99, to represent 100 we use second ~~dig~~ decimal digit followed by two first digits.

In binary system there are only two digits i.e 0 and 1. To express a number, the same decimal procedure is adopted here. to express 2 we use the second binary digit i.e 1 followed by first i.e 0. This gives 10 (read as one zero ~~no~~ and not ten) IIly 3 can be represented by 11 (one-one). to get 4 in binary system we used second digit followed by two first digits i.e (100)(one-zero-zero).

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001

Eg:- $1325 = 1000 + 300 + 20 + 5$

$= 1\times10^3 + 3\times10^2 + 2\times10^1 + 5\times10^0$

$(9)_{10} = (1001)_2$.

$(1001) = 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2$

$= 9.$

2 + 1 = 11.

→ Hexa decimal Number System :-

The word hexadecimal is composed of hexa meaning 6 and decimal meaning 10. The word hexadecimal means sixteen. its base & radix is 16. The maximum value of any digit in this system is 15, i.e 0, 2, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, A-10, B-11, C-12, D-13, E=14, F-15. after F we take , we take second digit followed by first digit,

③

Hexa decimal	Decimal.
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
A	10
B	11
C	12
D	13
E	14
F	15
10	16
11	17
12	18
13	19
14	20
15	21
16	22
17	23
18	24
19	25
1A	26
1B	27
1C	28
1D	29
1E	30
1F	31.

⑤

→ **Conversions :-**

1) Decimal to Binary conversion :-

Eg:- $11 = 8 + 0 + 2 + 1$

powers of $2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

⑥

coefficients = 1 0 1 1

$$(11)_{10} = (1011)_2 .$$

(or) Convert decimal 11 to its binary equalet :-
(double - Dabble method)

$11 \div 2 = 5$: remainder = 1 ↑ LSB
$5 \div 2 = 2$: remainder = 1
$2 \div 2 = 1$: remainder = 0
$1 \div 2 = 0$: remainder = 1 MSB

⇒ 1011 which is binary equalent of decimal 11.

(or) The above procedure can be expressed as

2 | 11
2 | 5 − 1
2 | 2 − 1
2 | 1 − 0
0 − 1

↑ $(1011)_2 = (11)_{10}$.

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Same OK.

→ Binary to Decimal Conversion :-

To conversion of a binary number in decimal equilen in Some what easy.

The decimal equilent is obtained by multiplying the individual digit by the ascending powers of 2 moving from right to left and then adding them. from example

$$1101 = 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$$
$$= 8 + 4 + 0 + 1$$
$$= 13.$$

⑧

Eg :- $0.1011 = 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} + 1\times2^{-4}$

$$= \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16}.$$

Next :- (See next page) $\frac{\overset{11}{8+0+2+1}}{16} = \frac{11}{16} = 0.6875$

1. Conversion of Hexa into Decimal No
2. Conversion of ~~Decimal~~ to. Hexa No follow

★ Then

→ Binary Arithmetic

(after those write this)

Binary Addition :- The complete binary addition table is as follows.

binary subtraction heads

rule 1 - 0+0 = 0 carry 0

rule 2 - 0+1 = 1 carry 0

rule 3 - 1+0 = 1 carry 0

rule 4 - 1+1 = 0 carry 1.

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Binary addition is carried out in the same manner as addition. The first three rules are OK. fourth one, when 1 is combined with 1 we get two which is represented as 10 in binary. We can say it is a zero with carry of 1. In binary 10 means two not ten.

Example :-

Binary	Decimal	Binary	Decimal	Binary	Decimal
110	6	1011	11	11	3
1011	9	111	17	101	5
1111	15	10010	18	1000	8

If A and B are two binary numbers, then their addition can be obtained by using the truth table given below.

Inputs		outputs	
A	B	Sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

⑨

→ Binary Substraction :-

The complete binary substraction table is as follows.

rule 1 – 0 − 0 = 0 No borrowing.
rule 2 – 1 − 0 = 0 " "
rule 3 – 1 − 1 = 0 " "
rule 4 – 0 − 1 = 1 with borrow of 1 from next higher column.

Examples :-

1101	(13)	1001	(9)	10111
1010	(10)	1010	(10)	101.11
0011	(3)	0001	(−1)	00100

Con

Let us explain the third Substraction $(11011)_2 - (10111)_2$

1) We Start from Right hand Side.
2) From the first column Substract 1−1=0 no barrow.
3) " " Second " " 1−1=0 " "
4) " " Third " " 0−1=1 with barrow 1

Here from the fourth column from left Side. leaving 0. Now 0−0=0 ∴ One has already borrowed with no borrowing.

5) From 5th column, Substract 1−1=0.

10

Test:-

add $(110011)_2$ to $(10.1101)_2$

Add 110, 111 and 101.

Substract $(0111)_2$ from $(1001)_2$

Substract $(111001)_2$ from $(1110001)_2$

~~Multiply $(10111)_2$ by $(101)_2$~~.

→ Conversion of Hexadecimal No into Decimal No:-

To convert a hexadecimal no into decimal no multiply each hexadecimal digit by its weight and add the resulting products.

Eg:-

Convert $(3A45)_{16}$ into decimal number.

$(3A45)_{16} = 3\times16^3 + 10\times16^2 + 4\times16^1 + 5\times16^0$

11

$= 3\times(4096) + 10\times(256) + 4\times(16) + 5\times(1)$

$= 12288 + 2560 + 64 + 5 = 14917.$

Hence $(3A45)_{16} = (14917)_{10}.$

Eg:- Convert $(5B6E)_{16}$ into decimal number.

$(5B6E)_{16} = 5\times16^3 + 11\times16^2 + 6\times16^1 + 14\times16^0$

$= 5\times4096 + 11\times256 + 6\times16 + 14\times1.$

$= 20480 + 2816 + 96 + 14 = 23406,$

$(5B6E)_{10} = (23406)_{10}.$

→ Conversion of Decimal no. into Hexadecimal No:-

To convert a decimal no into hexadecimal no the decimal no is divided by 16 and the remainders are converted to hex notations. Then we read from bottom to top to get hexadecimal no.

Eg:- Convert $(5000)_{10}$ in hexadecimal no.

16	5000	R	Hexanotation
16	312	8	8
16	19	8	8
16	1	3	3
	0	1	1

$= (1388)_{16}$,

⑫

Thus $(5000)_{10} = (1388)_{16}$.

→ Convertion of Hexadecimal into Binary no :-

To convert a hexadecimal no into a binary no Conve each hexadecimal digit to its 4-bit equivalent using the hex no.

Eg..- Convert $(7CF)_{16}$ into binary no.

$$\begin{array}{ccc} 7 & C & F \\ \downarrow & \downarrow & \downarrow \\ 0111 & 1100 & 1111 \end{array}$$

$(7CF)_{16} = (0111\ 1100\ 1111)_2$

C = 12 = 1100.

2 | 12
2 | 6 − 0
2 | 3 − 0
2 | 1 − 1
0 − 1

F = 15 = 1111

2 | 15
2 | 7 − 1
2 | 3 − 1
2 | 1 − 1
0 − 1

Eg :- Convert $(2B9D)_{16}$ into Binary number.

$$\begin{array}{cccc} 2 & B & 9 & D \\ 0010 & 1011 & 1001 & 1101 \end{array}$$

$(2B9D)_{16} = (0010\ \overset{1011}{\cancel{1100}}\ 1001\ 1101)_2$

→ 1's complemental Subtraction :-

Complement of a Number :-

In digital operation there are two types of complements of a binary no, which are used for complemental subtraction.

1) 1's complement :- This is also known as Radix-minus-one component. The 1's component of a binary no is obtained by changing its each zero (0) into ~~(1)~~ one (1) and each one (1) into zero (0).

13

for Eg :- 1's complement of $(1101)_2 = (0010)_2$.

2) 2's Complement :-

This is also known as true complement. The 2's complement of a binary number is obtained by adding one (1) to its 1's complement. Thus

2's complement = 1 + 1's complement,

Eg :- find out 2's Complement of $(1011)_2$,

we know that 1's complement of $(1011)_2 = (0100)_2$

Next we add 1 to this result,

Now we get $(0101)_2$.

$$\begin{array}{r} 0100 \\ 1 \\ \hline 0101 \end{array}$$

∴ 2's Complement of $(1011)_2 = (0101)_2$.

For 1's complemental subtraction the following procedure

① we first find the 1's complement of the subtrahend no. This is done by changing all its 1s to 0s and all its 0s to

2) Now we add this 1's complement to the minuend from which subtraction is required.

3) If there is a 1 carry in the most significant position of the result obtained by addition in step (ii) then this carry is removed and perform the end-around carry of the last 1.

4) If there is no-end-around carry (i-e 0 carry) then

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the answer is recomplemented and a -ve s
is attached to it to get the final result.

→ Subtract $(101)_2$ from $(111)_2$.

111 ← Minuend
+ 010 ← 1's complement of Subtrahend 101

$\boxed{1}$001
Add carry → 1 ← End round carry.

final result → 010

⑭

→ Subtract $(1101)_2$ from $(1010)_2$

1010 ← Minuend
0010 ← 1's complement of Subtrahend 1101

No carry → 1100
0011 ← As there is no end-around carry

Attach -ve − 0011 ← final result.

Subtract $(101101)_2$ from $(110011)_2$ using 1's complement method.

1's complement Method.

110011 ← Minuend
010010 ← 1's complement of (101101)

No carry $\boxed{1}$000101
1 ← end round carry

000110 ← Result.

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Direct Subtraction

```
    110011
(-) 101101
----------
    000110
----------
```

15

→ 2's complemental Substraction :-

For 2's complemental subtraction, we use the following procedure.

① change all 0s to 1s and 1s to 0s of Subtrahend to get the 1's complement and then add 1 to get 2's Complement.

② Now we add the 2's complemental to minuend.

③ If there is a carry in Result, it is dropped.

④ If carry is one (1), then the answer is +ve and needs no Recomplementing.

⑤ If there is no carry, we Recomplement the Result and attach a -ve sign to it.

① obtain 1's and 2's Complement of 1010101 and 0111000

i) Binary number = 1010101

1's complement = 0101010

2's complement = 1's Complement + 1

```
= 0101010
        1
  -------
  0101011
  -------
```

ii) Binary number = 0111000

1s Complement = 1000111

2's Complement = 1's Complement + 1

```
  1000111
        1
  -------
  1001000
```

→ ~~Boolean Algebra~~ :- Law

③ Use 2's Complemental to Subtract $(1101)_2$ from $(1010)_2$

0010 ← 1's complemental of 1101

1

0011 ← 2's Complental

Now

1010 ← Minuend.

0011 ← 2's complemental.

No carry 1101

Subtract 1 1

1100

Recomplement 0011

Put -ve -0011 ← final Result.

⑯

② Using 2's complemental, Subtract $(100111)_2$ from $(110011)_2$

011000 ← 1's complemental of 100111

1 ← Add one.

011001 ← 2's Complemental.

Now

110011 ← Minuend

011001 ← 2's Complemental

Carry $\boxed{1}$001100

Discard carry 001100 ← final Result.

Laws of Boolean Algebra:-

The logic operation involved in logic circuits & gates are mathematically described by Boolean algebra, that is invented by George Boole. Boolean algebra makes use of only two binary digits 1 and 0 which represents true & false. Only three operations are possible in Boolean algebra. They are.

① The OR addition (+)
② The AND multiplication (x)
③ The NOT operation

17

Laws of Boolean Algebra:-

(1) OR Laws:- In Boolean algebra (+) sign does not stand for addition as in ordinary mathematics.

In logic algebra symbolic logic the (+) sign indicates OR operation

A+B=C means that if A is true OR B is true, then C will be there.

Accordingly A+B=C may equivalent be expressed as

$A \cup B = C$ & $A \vee B = C$.

The laws of OR operation may be expressed as

$0+0=0$

$0+1=1$

$1+0=1$

$1+1=1$

These equations imples that o/p is zero only when both inputs are zero, while the meaning of last 3 eq's is that the o/p is 1 when either input A & B & both are 1.

The OR laws in general form may be expressed as

$A+0=A$ ——— ①

$A+1=1$ ——— ②

$A+A=A$ ——— ③

$A+\overline{A}=1$ ——— ④

Where $\overline{A}$ is complement & -ve of A.

② **AND laws:-** In boolean algebra the AND operation, represented by multiplication. The eq $A \cdot B = C$ means that 1) output C is 1 only when both A and B are 1.

2) o/p C is true only when both A and B are true

The laws of AND operation may be expressed as

$0 \cdot 0 = 0$.
$0 \cdot 1 = 0$.
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$.

The AND laws in general may be expressed as

$A \cdot 0 = 0 \longrightarrow$ ⑤
$A \cdot 1 = A \longrightarrow$ ⑥
$A \cdot A = A \longrightarrow$ ⑦
$A \cdot \overline{A} = 0 \longrightarrow$ ⑧

③ **NOT laws:-** It is a complementation operation and its symbol is an overbar. The Laws of NOT operation are

$\overline{0} = 1 \longrightarrow$ ⑨
$\overline{1} = 0 \longrightarrow$ ⑩
$\overline{A} = A \longrightarrow$ ⑪

④ **Commutative laws:-**

$A + B = B + A \longrightarrow$ ⑫

and $A \cdot B = B \cdot A \longrightarrow$ ⑬

These laws show that the order, in which a combination of terms is preformed. does not affect the final result.

⑤ Associative Laws:-

$A + (B+C) = (A+B) + C \longrightarrow$ ⑭

$A \cdot (B \cdot C) = (A \cdot B) \cdot C \longrightarrow$ ⑮

⑥ Distributive Laws:-

$A(B+C) = AB + AC \longrightarrow$ ⑯

$(A+B) \cdot (A+C) = A + BC. \longrightarrow$ ⑰

⑦ ~~Ab~~ Absorptive Laws

⑲

$A + AB = A$ ———— 18

$A \cdot (A+B) = A$ ———— 19

$A \cdot (\overline{A}+B) = AB$ ———— 20.

⑧ DeMorgan's Theorems:-

① $\overline{A+B} = \overline{A} \cdot \overline{B}$

② $\overline{A \cdot B} = \overline{A} + \overline{B}$

Convert $(2047)_{10}$ to hexadecimal number

16	2047	R	Hex notation
16	127	15	F
16	7	15	F
16	0	7	7

20

∴ $(2047)_{10} = (7FF)_{16}$.

→ Conversion of Hexadecimal into Binary Number :-

To convert a hexadecimal no into a binary no convert each hexadecimal digit to its 4-bit equivalent using the hex no.

Eg convert $(7CF)_{16}$ into binary number.

we

$$\begin{array}{ccc} 7 & C & F \\ \downarrow & \downarrow & \downarrow \\ 0111 & 1100 & 1111 \end{array}$$

$(7CF)_{16} = (0111\ 1100\ 1111)_2$.

C : 12 : 1100

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 - 0 \\ 2 & 3 - 0 \\ 2 & 1 - 1 \\ & 0 - 1 \end{array}$$

F : 15 : 1111

$$\begin{array}{r|l} 2 & 15 \\ 2 & 7 - 1 \\ 2 & 3 - 1 \\ 2 & 1 - 1 \\ & 0 - 1 \end{array}$$

Convert $(2B9D)_{16}$ into Binary number.

$$\begin{array}{cccc} 2 & B & 9 & D \\ 0010 & 1011 & 1001 & 1101 \end{array}$$

$(2B9D)_{16} = (0010\ 1011\ 1001\ 1101)_2$.

Now almost all computer use hexadecimal system because this representation requires less space as compared to Binary and octal system.

→ Basic Logic Gates :-

circuits which are used to process digital signals are called logic gates. Gate is a digital ckt ~~tho~~ with one & more input voltages but only one output voltage. The most basic gates are called the AND gate the OR gate and NOT gate.

Logic gates are two types. Combinationa and sequential. In combinational gates, the output at any instant depends upon the inputs at that instant. But in case of sequential gates the output depends upon the order & sequence in which the inputs are applied. Hence sequencial gates possess a memory function. OR, NOT, NAND, NOR, XOR are examples of combinational gates. Flip~~s~~ Flops, Counters and Registers form the sequential gates.

For analysis and design of digital ckt, symbolic logic was invented by Boole and is called the Boolean Algebra. Here only two states are possible true (or) false. And only three operations are possible in Boolean algebra and they are

1) The OR addition (+)
2) the AND multiplication (x)
3) the NOT operation represented by a bar over the variable.

OR (Addition) gate.

An OR gate has two & more input signals but only one output signal. It is called an OR gate because the output voltage is high if any & all of the input voltages are high. The symbolic representation of a two input OR gate as shown in fig ⓐ and ⓑ

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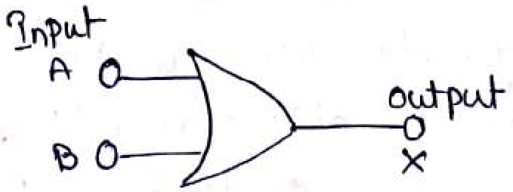
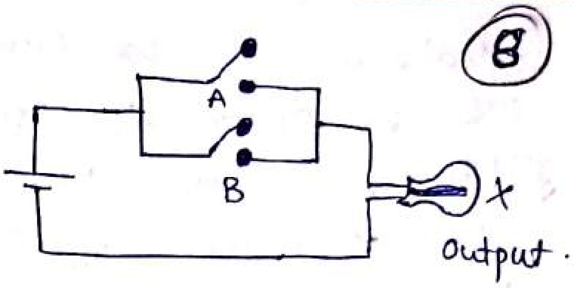




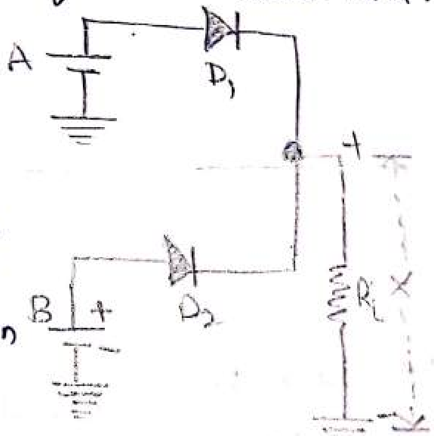
Switches are connected in parallel with each other. Lamp will glow when A & B & both Switches are closed.

OR gate represented by eq $\boxed{A + B = X.}$

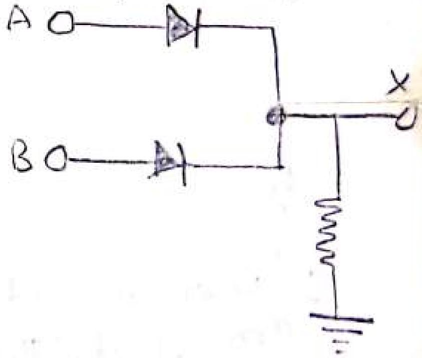
Truth Table:- A truth table is a table that shows all the input output possibilities of a logic CKT. fig 2a shows the truth table of a two input OR gates. (2b) shows one way to build a two input OR gate. fig (2C) is the simplified form of the same CKT.

Input		output	
A	B	X = A+B	
0	0	0	
0	1	1	Lamp on
1	0	1	Lamp on
1	1	1	Lamp on

(2a)



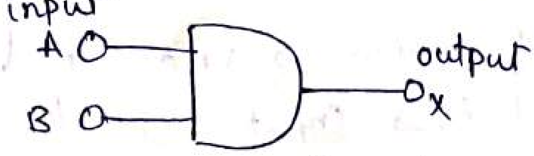
(2b).



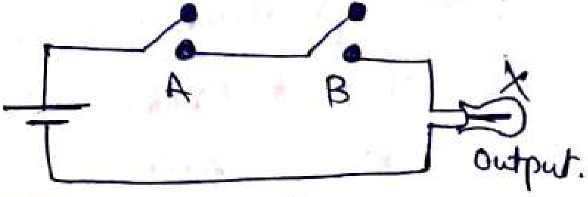
22

→ AND Multiplication:-

The AND gate is a circuit which provides an o/p when all the i/p are simultaneously present. The symbolic representation of a two i/p AND gate is shown in fig ⓐ, ⓑ shows its electrical equivalent.



ⓐ Symbol



electrical

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Mathematically we can write $A \cdot B = X$

The dot placed b/w input symbols will give the AND gate. When A is closed (true) and B is also closed (~~true~~ (true) then X is true. The state closed & true can be written by no 1 and the false state (open state) can be written by no 0. We have

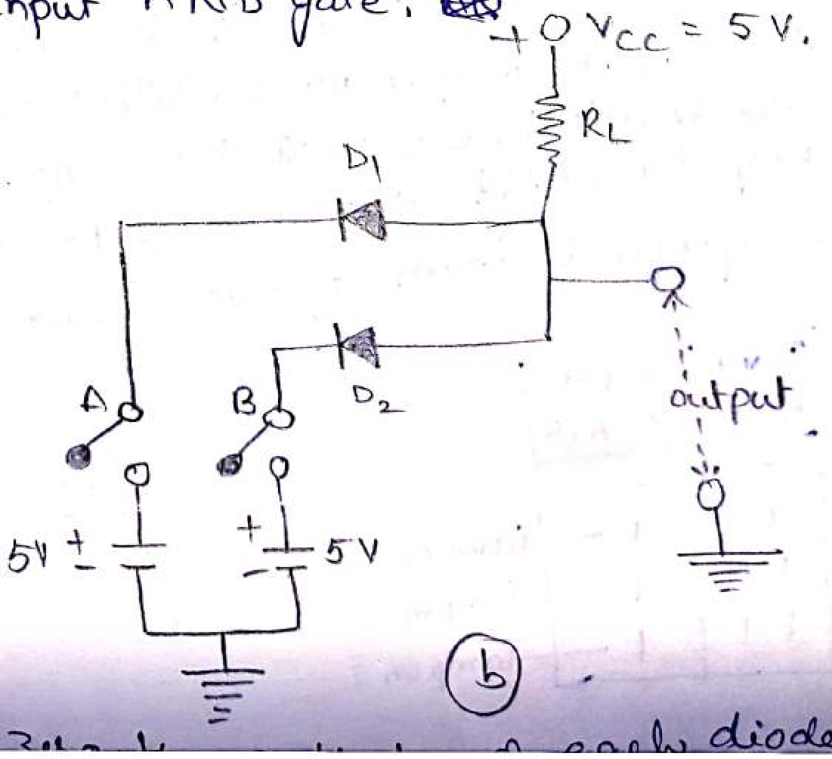
$1.1 = 1$ and 1.0 & $0.1 = 0$.

23

Truth table :- fig ⓐ summarizes all input-output possibilities for a 2-input AND gate.

Input		output
A	B	$X = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

ⓐ



ⓑ

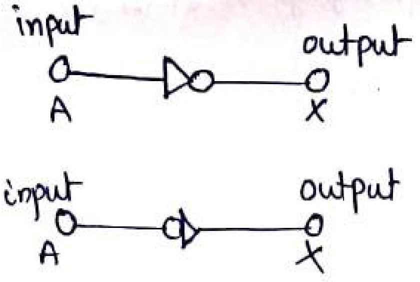
ⓑ ↓

① When A and B are each diode is

→ The NOT circuit :-

The NOT gate is a gate with only one input and one output. It is called so because the output state is always opposite to that of input state. That is when the input voltage is high, the output is low and vice-versa. The NOT gate is also called an inverter fig a shows the symbol ~~of fr~~ the NOT gate.

(24)



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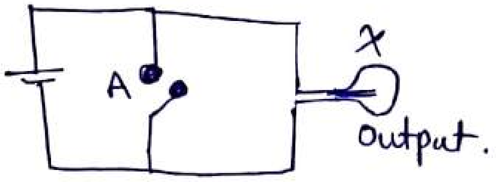


ⓑ

fig ⓑ shows the electrical analog of NOT gate. Here when switch A is closed (i/p high) the bulb will not glow.

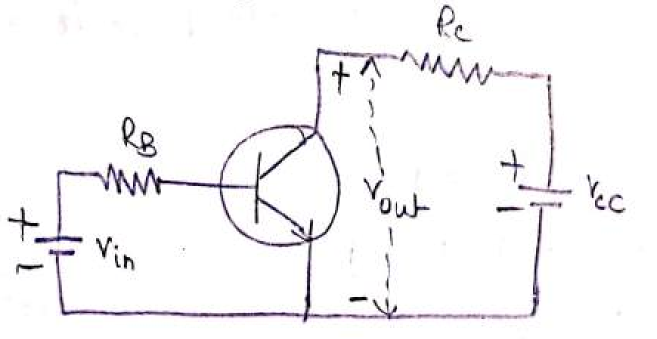
Truth table :- fig (a and b) shows the truth table of NOT gate.

Input	output
A	X = $\bar{A}$
0	1
1	0

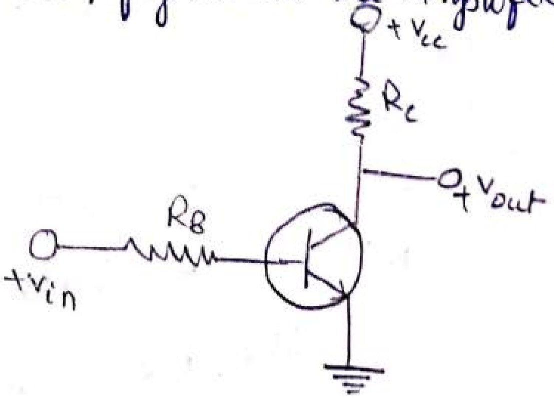
&

Input	output
A	X = $\bar{A}$
Low	High
High	Low

circuit operation :- fig ⓒ shows the common-emitter ckt which is used as a NOT gate. fig ⓓ is the simplified version of ⓒ



ⓐ

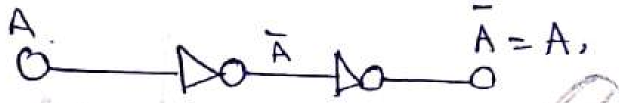


NOT operation

The NOT operation is represented by a line drawn over the top of the input. The negation of A is $\overline{A}$, called NOT-A Some times A-bar.

$$\boxed{\overline{0} = 1 \text{ and } \overline{1} = 0 \text{ Also } \overline{\overline{A}} = A}$$

The third Law can be written following the definition of an inverter $\overline{\overline{A}} = A$ can be verified by fig, A is inverted twice $\overline{\overline{A}}$ and is indentical to A.



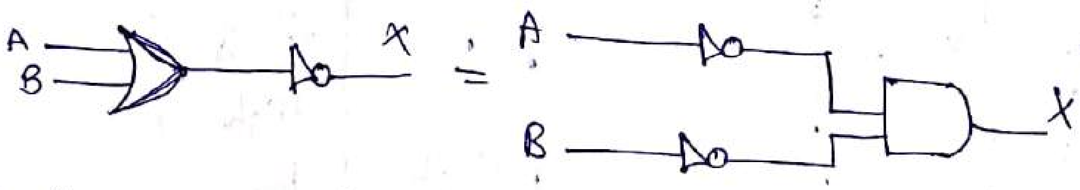
25

→ DE Morgan's Theorem:-

Theorem:-① The Complement of the Sum of two & more Variables is equal to the product of the complement of the Variables i.e for two inputs gate we can write

$\overline{A+B} = \overline{A} \cdot \overline{B}$

De Morgan's first theorem can be represented by fig.



Proof:- Here we are going to prove that $\overline{A+B} = \overline{A} \cdot \overline{B}$. Since a variable can have only two values (0 & 1) and there are two variables in the equation. we have four cases.

1) When $A = 0$ and $B = 0$

LHS $= \overline{A+B} = \overline{0+0} = \overline{0} = 1$

RHS $= \overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{0} = 1 \cdot 1 = 1$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$.

2) When A = 0 and B = 1

L.H.S $= \overline{A+B} = \overline{0+1} = \overline{1} = 0$

R.H.S $= \bar{A}\cdot\bar{B} = \bar{0}\cdot\bar{1} = 1\cdot 0 = 0$.

Hence $\overline{A+B} = \bar{A}\cdot\bar{B}$

3) When A = 1 and B = 0

LHS $\overline{A+B} = \overline{1+0} = \overline{1} = 0$

RHS $= \bar{A}\cdot\bar{B} = \bar{1}\cdot\bar{0} = 0\cdot 1 = 0$.

Hence $\overline{A+B} = \bar{A}\cdot\bar{B}$

26

4) When A = 1 and B = 1

LHS $= \overline{A+B} = \overline{1+1} = \overline{1} = 0$.

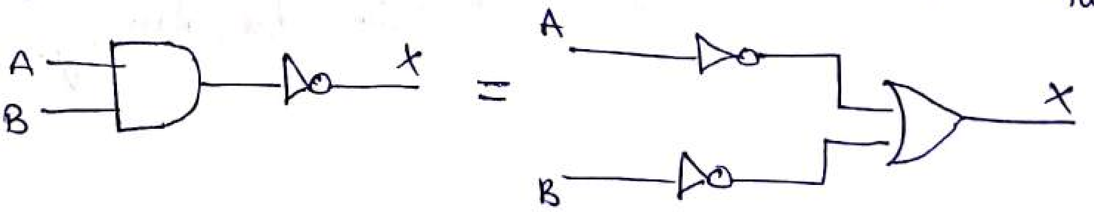
RHS $= \bar{A}\cdot\bar{B} = \bar{1}\cdot\bar{1} = 0\cdot 0 = 0$.

Hence $\overline{A+B} = \bar{A}\cdot\bar{B}$.

Hence we see that in every case left hand side (L.H.S) is equal to right hand side (RHS), therefore the theorem is proved.

Theorem: 2 :- The complement of the product of two or more variables is equal to the sum of the complements of variables i-e $\overline{A\cdot B} = \bar{A} + \bar{B}$

fig ⑥ shows the representation of De Morgan's second theorem.



Here we have to prove that $\overline{A\cdot B} = \bar{A} + \bar{B}$ in the previous theorem. there are only four cases possible.

1) A = 0, B = 0

LHS $= \overline{A\cdot B} = \overline{0\cdot 0} = \bar{0} = 1$

RHS : $\bar{A} + \bar{B} = \bar{0} + \bar{0} = 1 + 1 = 1$

Hence $\overline{A\cdot B} = \bar{A} + \bar{B}$

2) $A = 0, B = 1$

LHS $= \overline{A\cdot B} = \overline{0\cdot 1} = \overline{0} = 1$

RHS $= \overline{A} + \overline{B} = \overline{0} + \overline{1} = 1 + 0 = 1$

Hence $\overline{A\cdot B} = \overline{A} + \overline{B}$

3) $A = 1, B = 0$

LHS $= \overline{A\cdot B} = \overline{1\cdot 0} = \overline{0} = 1$

RHS $= \overline{A} + \overline{B} = \overline{1} + \overline{0} = 0 + 1 = 1$

Hence $\overline{A\cdot B} = \overline{A} + \overline{B}$

4) $A = 1, B = 1$

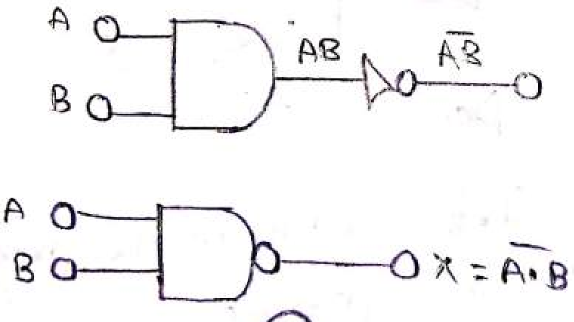
LHS $= \overline{A\cdot B} = \overline{1\cdot 1} = \overline{1} = 0$

RHS $= \overline{A} + \overline{B} = \overline{1} + \overline{1} = 0 + 0 = 0$

Hence $\overline{A\cdot B} = \overline{A} + \overline{B}$

→ NAND Gate:-

This circuit has become So popular that the words NOT-AND have been abbreviated as NAND gate. So NAND gate is a Combination of AND gate and a NOT gate. The Symbol of a NAND gate is shown in fig ⓐ Here the bubble on the output reminds us of the inversion after ANDing.



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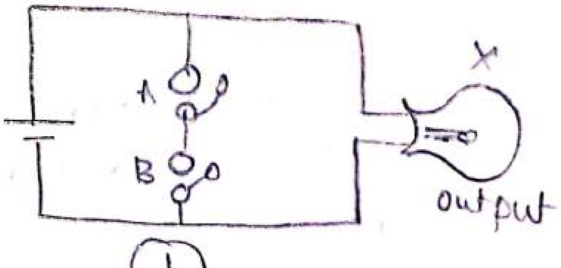


ⓑ

fig ⓑ shows the electrical analog of NAND gate. As the o/p is zero the lamp will not glow when switch A and B are closed ($A = B = 1, X = 0$) Hence the truth table is follows.

28

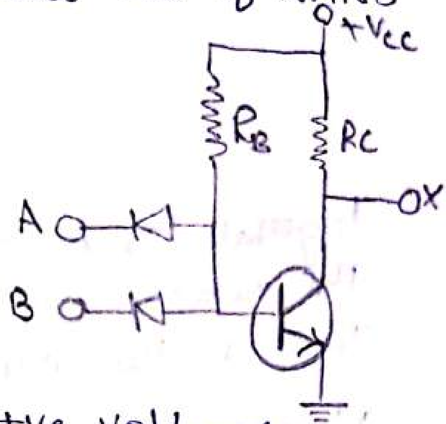
Input		Output
A	B	$X = \overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

1) When $A=0$ $B=0$ then $AB=0$ and $\overline{AB}=1$

2) When $A=0, B=1$ then $AB=0$ and $\overline{AB}=1$

3) When $A=1, B=0$. then $AB=0$ and $\overline{AB}=1$

4) When $A=1, B=1$ then $AB=1$ and $\overline{AB}=0$



The output of NAND gate is given by $X = \overline{A.B}$ fig © shows ckt of NAND gate. Here diode AND ckt is connected to a transistor NOT ckt which gives the NAND ckt. from this when the two inputs are zero, the transistor will not conduct. So the output will be +Vcc

On the other hand when +ve voltages are applied to the two inputs the transistor will conduct into saturation region. Now the output will drop to zero. In this way the ckt behaves as a reverse AND ckt. and hence called as NAND ckt. We know OR and AND gates can be built by using diodes or transistors but a NOT gate can be built by using transistors only. Hence a NAND gate can be built by using diode transistor logic (DTL) & transistor-transistor logic (TTL).

→ NAND gate is a Universal Gate:-

The NAND gate is called a universal gate since any logic ckt can be built by using NAND gate only. The other example of a universal gate is a NOR ~~as~~ gate as ~~shown~~ in fig@ shows how NAND gate can be used as NOT gate, AND gate and OR gate respectively. When all the inputs of a NAND gate are connected

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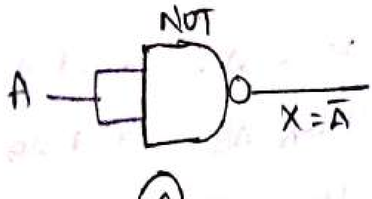

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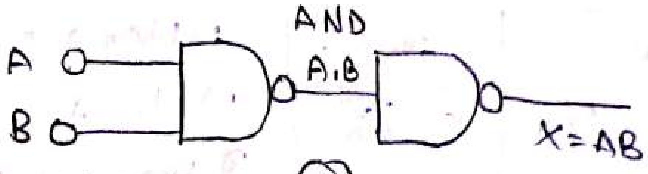

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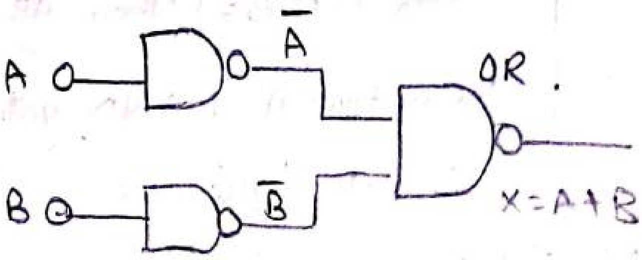
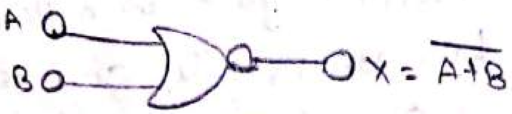


ⓒ

together the resulting ckt is a NOT ckt. The reason is as follows:-
If A =1, the o/p of NAND gate is $\overline{A\cdot A} = \overline{1\cdot 1} = \overline{1} = 0$
If A=0 the output of NAND gate is $\overline{A\cdot A} = \overline{0\cdot 0} = \overline{0} = 1$.
~~If use~~ The use of two NAND gates to produce an AND gate is shown in fig ⓑ. llly fig ⓒ shows how OR gate can be made out of three NAND gates.

→ **NOR gate:-** In this logic ckt NOT gate followed a OR gate. This is also popular like the NAND gate and the words NOT-OR have been abbreviated as NOR gate. The symbol of a NOR gate is shown in fig ⓐ. As in the NAND gate symbol the bubble in the output signifies that invertion



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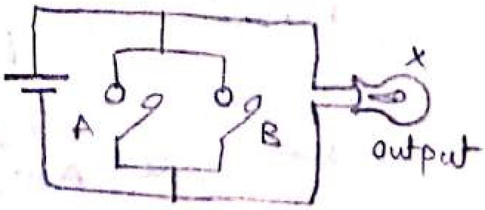

ⓑ

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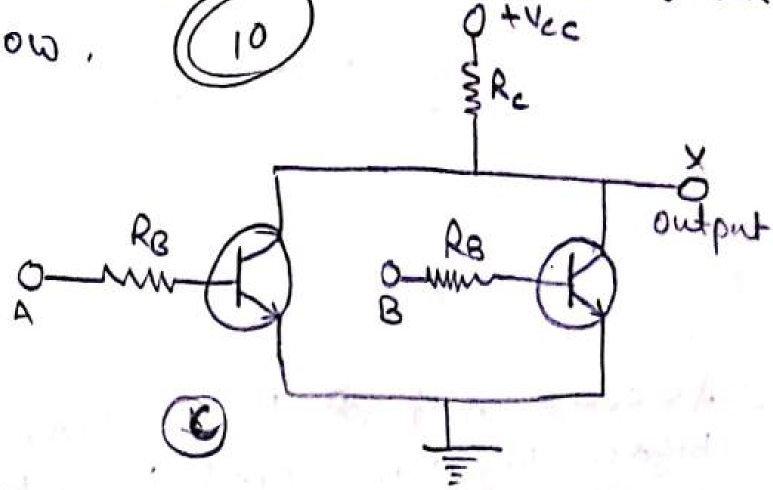
takes place after the OR Ring fig ⓑ shows the electrical analog of NOR gate. As from this fig the lamp will not glow. i-e output is zero when either of the inputs A & B is high (A, B switch is closed). The truth table of NOR gate is shown in below. ⑩

Input		Output
A	B	$X = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0



ⓒ

1) when A = 0, B = 0 then $A+B = 0$
$X = \overline{A+B} = 1$

2) When A = 0, B = 1 then $A+B = 1$
$X = \overline{A+B} = 0$

3) when A = 1, B = 0 then $A+B = 1$
$X = \overline{A+B} = 0$

4) When A = 1, B = 1 then $A+B = 1$
$X = \overline{A+B} = 0$

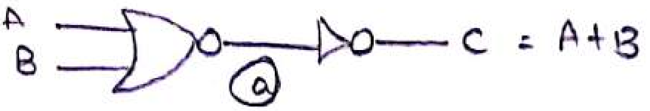
㊿

The output of a NOR gate can be written as $X = \overline{A+B}$ as shown ~~in fig~~ the ckt of a NOR gate. The output x is 1 only when both transistors are cutoff. When A = 0 and B = 0 For any other condition of input like A = 1, B = 0 & A = 0, B = 1, & A = 1 and B = 1 one & both transistors saturate and the result is that output x is 0.

→ NOR gate is a universal gate:-

NOR gate is a universal gate because it can be used to perform the basic logic functions OR, AND and NOT. fig ⓐ shows how a NOR gate can be used as OR gate, AND gate and NOT gate.

As shown in fig ⓐ the output of NOR gate is $\overline{A+B}$. Now using another inverter in the output, the output is inverted and beccome $C = A+B$. This is logic function of a normal OR gate. So NOR gate can be used as OR gate.



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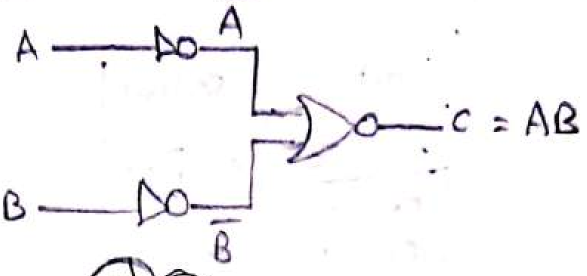


ⓑ

3) As shown in fig ⓒ two inverters are used for each input before they are applied to a NOR gate. The output of two inverters is $\bar{A}+\bar{B}$. When this is applied to NOR gate, the final output becomes AB.

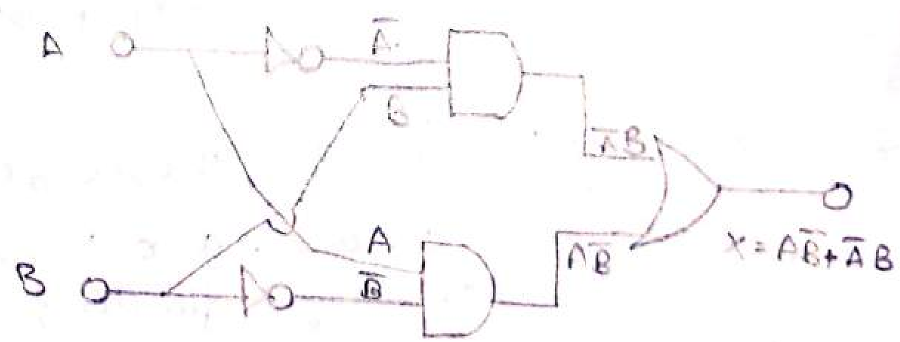
As Shown in fig ⓑ the two input have been tied-together. The output is $\overline{A+A}$. This is equal to $\bar{A}$ (using Demorgan theorem). So, a NOR gate can be used as NOT gate.

→ Exclusive OR gate (XOR gate)

The symbol of a XOR gate is shown in fig ⓐ. fig ⓑ shows the ckt of a XOR gate. There are two inputs and one output. The exclusive-OR operation is denoted by ⊕, Hence the output is given by $X = A \oplus B = A\bar{B} + \bar{A}B$.

$X = A \oplus B$

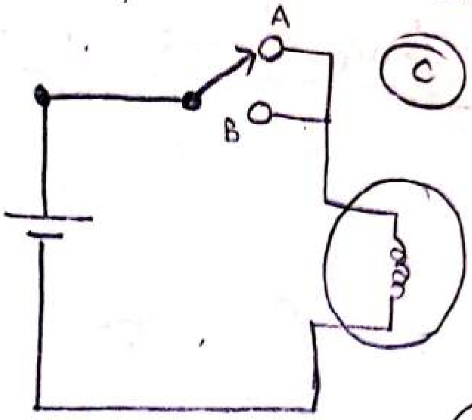
ⓐ



ⓑ

$X = A \oplus B = A\bar{B} + B\bar{A}$

The CKt of XOR gate is shown in fig ⓒ. The bulb will glow when A=1 or B=1 but not both. The Boolean expression for XOR gate is expressed as



ⓒ

$$X = A \oplus B$$
$$= (A+B)\,\overline{AB}$$
$$= A\overline{B} + \overline{A}B.$$

④

Truth table:- The truth table of a XOR gate can be developed as follows.

1) when A=0, B=0, $X = \overset{A\cdot\overline{B}}{0\cdot 1} + \overset{\overline{A}\cdot B}{1\cdot 0} = 0+0 = 0$

2) when A=0, B=1, $X = 0\cdot 0 + 1\cdot 1 = 0+1 = 1$

3) when A=1, B=0 $X = 1\cdot 1 + 0\cdot 0 = 1+0 = 1$

4) when A=1, B=1, $X = 1\cdot 0 + 0\cdot 1 =$ ~~0+0 exp~~. $0+0=0$

32

The truth table of XOR gate is given below.

input		out put
A	B	$X = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

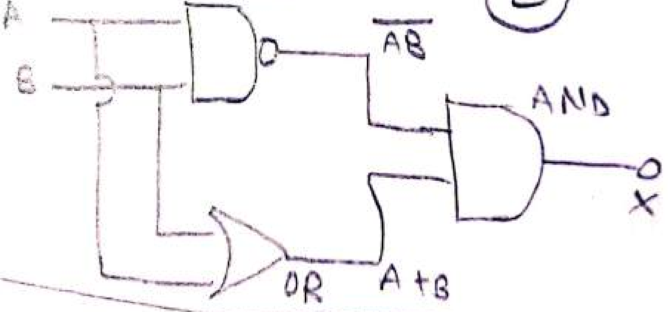
Inputs		$A\overline{B}$	$\overline{A}B$	out put
A	B			$X = A\overline{B} + \overline{A}B$
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0

from the truth table the output is 1 only when the inputs are different.

XOR operation is also called mod-2 addition and rules of addition are

$0 \oplus 0 = 0$, $0 \oplus 1 = 1$

$1 \oplus 0 = 1$ $1 \oplus 1 = 0$.



ⓓ

from above we have conclude that mod-2 addition is a binary addition in which the carry is neglected. The other way to implement XOR is shown in fig ⓓ.

Applications of XOR gate:-

1) XOR ~~is used~~ gate can be used as Binary to Gray code converter
2) It can be used as a parity checker
3) It is also used in Half adder ckt.

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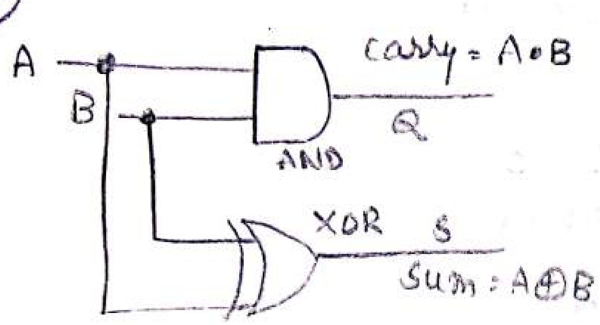
Half Adder :-

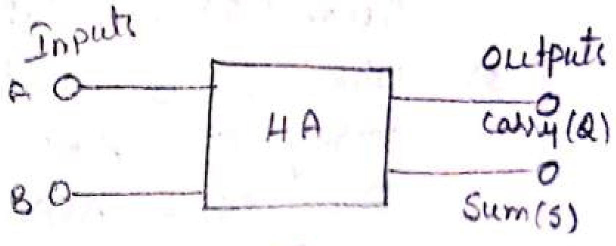
A logic ckt that adds two bits producing a sum and a carry to be used in the next higher position is called a half-adder. In a half adder, there is no possibility for a carry input. The previous carry input is denoted by Q'. ~~A~~ half adder can be used for binary addition of lower most bit called as least significant bit.

The ckt is called as half adder because it cannot accept a carray (Q') from previous addition. To accept a previous carray (Q') a full adder is used.

34



ⓐ



ⓑ

Half adder in capable of combining two binary no. and produces an output which contains sum and carry. Fig (2a) shows the ckt of a half adder. It consists of an exclusive-OR gate and an AND gate. The output of the exclusive-OR gate is called the SUM, while the output of the AND gate is called the ~~ca~~ CARRY.

Fig ⓑ shows the symbol of a half-adder. The AND gate produces a high output only when both inputs are high. The truth table of a Half-Adder is developed by writing the truth table output of AND gate in carry column and the output truth table of ex-OR gate in SUM column. The ckt is called half adder because it cannot accept a carry in from previous addition. For this purpose, we need a 3-i/p adder called full adder.

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Input		output	
A	B	CARRY (Q)	Sum (S)
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0
		and gate o/p	XOR gate o/p

$\overset{0}{A} \oplus \overset{1}{B} = \bar{A}\bar{B} + \bar{A}B$
$= 0.0 + 1.1$
$= 0 + 1 = 1$

The explanation is as follows:-

1) When $A = 0$ and $B = 0$
carry $(Q) = A \cdot B = 0 \cdot 0 = 0$.
Sum $(S) = A \oplus B = 0 \oplus 0 = 0$

2) when $A = 0$ and $B = 1$
carray $(Q) = A \cdot B = 0 \cdot 1 = 0$
Sum $(S) = A \oplus B = 0 \oplus 1 = 1$

35

3) When $A = 1$ and $B = 0$
carry $(Q) = A \cdot B = 1 \cdot 0 = 0$
Sum $(S) = A \oplus B = 1 \oplus 0 = 1$

4) When $A = 1$ and $B = 1$
carry $(Q) = A \cdot B = 1 \cdot 1 = 1$
Sum $(S) = A \oplus B = 1 \oplus 1 = 0$

→ Full Adder:-

For higher-order colums the half adder ckt cannot be used for binary addition. For this purpose we need a ckt called full adder. having 3 i/p's (two inputs for the two bits to be added ~~as a to~~ and the third bit is the carray from a lower column).

Thus full adder as a logic ckt that adds three bits - two bits to be added and a carry bit from previous addition which results

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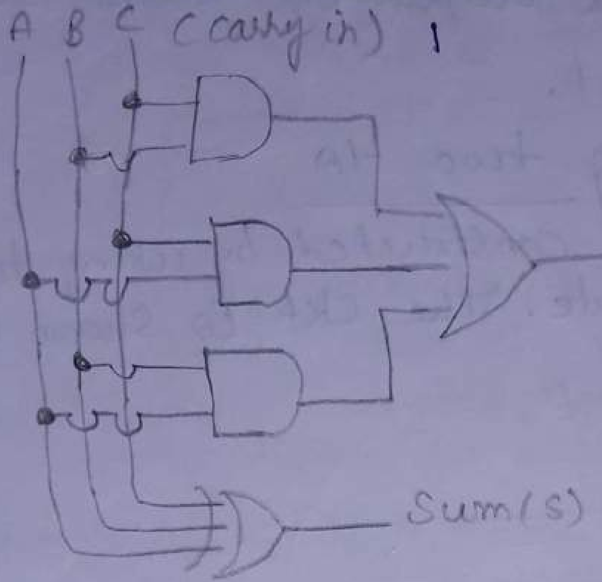
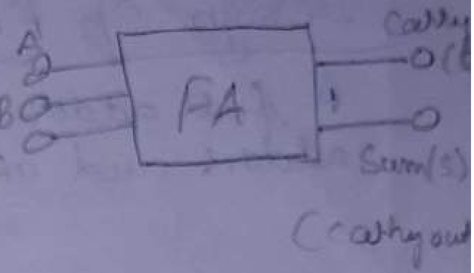
is a Sum and carry. A Simple ckt is shown in fig (a) and (b) showns the Symbol of a full adder. It has two i/p's A and B, plus a third input C. I/p C is also called the ~~Carray in~~ CARRY IN as it comes from a lower-order column.

(13)



(a)



36

~~They~~ There are two outputs Sum and carry. The o/p carry is also called the CARRY OUT as it goes to the next higher column. The Truth table of a FA has also been given. It can be easily verified. Let us for example consider the lowest entry.

Truth table of FA				
Input			output	
A	B	C	Carry (Q)	Sum (S)
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

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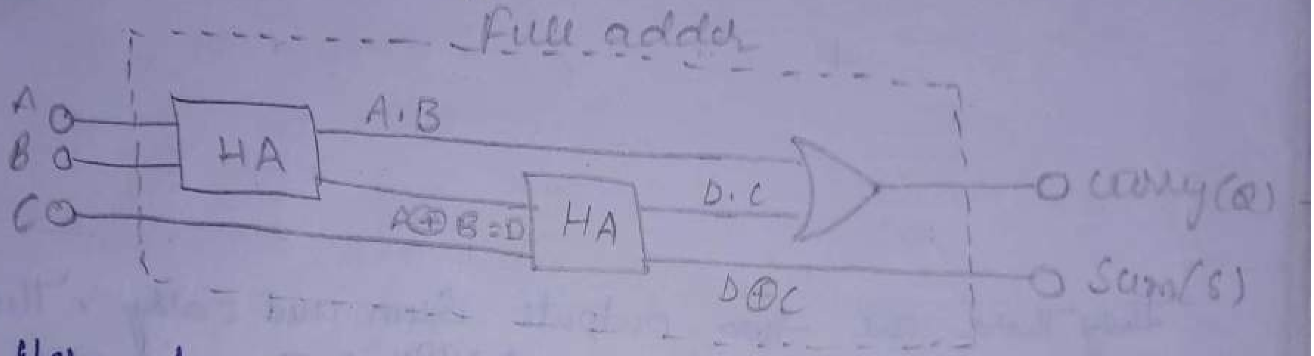
Here inputs are A = 1 , B = 1, and C = 1
C = 1 signifies that a carry had been generated
the previous column addition.
Now 1+1 = 1 i.e a 1 with a carry of 1. The same
is the case with the output in truth table where
sum = 1 and carry = 1.

Design of FA using two HA

A full adder can be constructed by using two half adders and an OR gate. The ckt is shown in fig.



Here also consider the case when A = 1, B = 1 and c = 1

$A \cdot B = 1 \cdot 1 = 1.$

$D = A \oplus B = 1 \oplus 1 = 0$

$D \cdot c = 1 \cdot 0 = 0$

$D + c = 0 \oplus 1 = 1$

carry = $A \cdot B + D \cdot C = 1 + 0 = 1.$

Sum = $D \oplus c = 1.$

Hence we can say that the above ckt will function as a FA.

(37)

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C_3 ~~The third adder performs 0+0+1 carry =1 with carry 0. The fourth adder adds 1+1+0 carry = sum 0 with carry 1 (both of which appears on display unit) Therefore the final number appears as 10101.~~

parallel adder

By cascading, a no. of full adders, we can get an n bit binary adder ckt which is generally called a parallel adder.

Suppose we require to add 4-bit binary numbers A_4, A_3, A_2, A_1 and B_4, B_3, B_2, B_1 then we get a sum S_4, S_3, S_2, S_1 such that

$$\begin{array}{rrrrr} & A_4 & A_3 & A_2 & A_1 \\ + & B_4 & B_3 & B_2 & B_1 \\ \hline S_5 & S_4 & S_3 & S_2 & S_1 \end{array}$$

58

where S_5 indicates overflow carry if the sum exceeds four bits. for adding them (two 4-bit no's) we need 4 full adders (FA) connected in parallel. the first adder could be half adder (HA) because only two bits are to be added and there is no carry, but all subsequent columns need full adder because we have to handle three bits at a time (two binary digits and a carry generated by the previous column) fig shows a parallel four-bit binary adder.